Examen maxplus, ENSTA-Bretagne, **ENSI 2**. 18 mars 2016,

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We consider a public transport system of buses with 4 lines and 4 buses. There are only two stations where travelers can change lines. This system can be represented by a Petri net (see Figure 1). Each token corresponds to a bus. The places p_1, p_2, p_3, p_4 represent the lines. These places are composed of a number which corresponds to the minimum amount of time that the token must remain in its place (this corresponds to the transit time). The transitions t_1, t_2 ensure synchronization. They are only crossed when each upstream place of the transition has at least one token which has waited sufficiently long. In this case, the upstream places lose a token and the downstream places gain one. This structure ensures that the correspondence will be performed systematically and that the buses will leave in pairs.

1. Let us assume that at time $t = 0$, the transitions t_1 and t_2 are crossed for the first time and that we are in the configuration of Figure 1 (this corresponds to the initialization). Give the crossing times for each of the transitions.

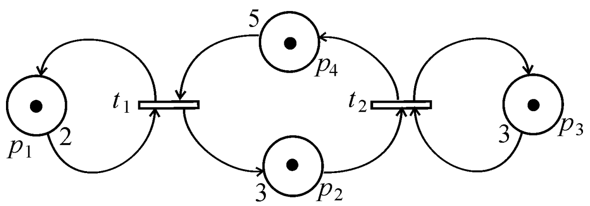

Figure 1: Petri net of the bus system

2. Let us denote by $x_i(k)$ the time when the transition t_i is crossed for the k^{th} time. Show that the dynamics of the model can be written using states :
$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$$
where $\mathbf{x} = (x_1, x_2)^{\mathrm{T}}$ is the state vector. Remember that here k is not the time, but an event number.

3. Let us now attempt to reformulate elementary algebra by redefining the addition and multiplication operators as follows :
$$\begin{cases} a \oplus b = \max(a, b) \\ a \otimes b = a + b \end{cases}$$
Thus $2 \oplus 3 = 3$ whereas $2 \otimes 3 = 5$. Show that in this new algebra (called max-plus), the previous system is linear.

4. Define $y(k) = x_1(k) + 2x_2(k)$. With Python, simulate the system for $k \leq 10$.

5. Assume that we do not know $\mathbf{x}(k)$ anymore. Instead, we know the state equation obtained at Question 2 and the output $y(k)$ obtained at Question 4. Using PyIbex and Vibes, make a program which computes (and draw) for all k the set $\mathbb{X}(k)$ of all feasible state vectors $\mathbf{x}(k)$.
