Examen localisation, UV 3.7, ENSI 2.

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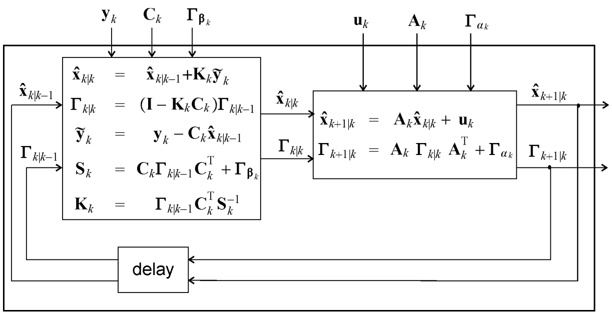
Le mardi 8 janvier 2019. Appareils électroniques interdits.
Polycopié et notes manuscrites autorisés
Durée: 60 minutes.

Consider a vehicle (a robot) moving in a plane and described by the following discretized state equations

$$\left\{\begin{array}{lll} x_1(k+1) & = & x_1(k)+v(k)\cos(\theta(k))+\alpha_1(k) \\ x_2(k+1) & = & x_2(k)+v(k)\sin(\theta(k))+\alpha_2(k) \end{array}\right.$$

where the $\alpha_i(k)$ are white unit centred Gaussian noise, *i.e.*, $\alpha_i(k) : \mathcal{N}(0,1)$. The vector $(x_1(k), x_2(k))$ represents the coordinates of the center of the vehicle at time k, $\theta(k)$ is the heading, $v(k)$ is its speed. The sampling time is taken as $dt = 1sec$. Both $v(k)$ and $\theta(k)$ are assumed to be known exactly. At each k, a GPS provides a measurement on $\mathbf{x}$ with an accuracy of $10m$.

1) We want to use a Kalman filter (see the figure) in order to estimate the state $\mathbf{x}_k$ with a higher precision than that given by the GPS.



Explain how this can be done and explain how you choose the values for the inputs $\mathbf{u}_k, \mathbf{y}_k, \mathbf{A}_k, \mathbf{C}_k, \mathbf{\Gamma}_{\boldsymbol{\alpha}}, \mathbf{\Gamma}_{\boldsymbol{\beta}}$ of the Kalman filter. Explain also how the filter can be initialized.

2) We now assume that the speed is not measured anymore, but only its derivative $a(k)$. More precisely, we assume that we have

$$v(k+1) = v(k) + a(k) + \alpha_v(k)$$

where $\alpha_v(k)$ is white unit centred Gaussian noise. Explain in details how the Kalman filter can be used in this context to get a precise localization.

Correction de l'examen du 8 janvier 2019

1) Our system can be written under the form:

$$\left\{\begin{array}{lcl} \mathbf{x}_{k+1} & = & \underbrace{\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right)}_{\mathbf{A}_k}\mathbf{x}_k + \underbrace{\left(\begin{array}{c} v(k)\cos(\theta(k)) \\ v(k)\sin(\theta(k)) \end{array}\right)}_{\mathbf{u}_k} + \boldsymbol{\alpha}_k \\ \mathbf{y}_k & = & \underbrace{\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right)}_{\mathbf{C}_k}\mathbf{x}_k + \boldsymbol{\beta}_k \end{array}\right.$$

Moreover

$$\boldsymbol{\Gamma}_{\boldsymbol{\alpha}_k} = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) \text{ and } \Gamma_{\beta_k} = \left(\begin{array}{cc} 10^2 & 0 \\ 0 & 10^2 \end{array}\right).$$

For the initialization, we can take

$$\hat{\mathbf{x}}_{0|-1} = (0\ 0)^{\mathrm{T}} \text{ and } \boldsymbol{\Gamma}_{0|-1} = 10^{10} \cdot \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right).$$

2) We add a state variable $x_3 = v$. The system is now described by:

$$\left\{\begin{array}{lcl} \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right)(k+1) & = & \underbrace{\left(\begin{array}{ccc} 1 & 0 & \cos(\theta(k)) \\ 0 & 1 & \sin(\theta(k)) \\ 0 & 0 & 1 \end{array}\right)}_{\mathbf{A}_k} \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right)(k) + \underbrace{\left(\begin{array}{c} 0 \\ 0 \\ a_k \end{array}\right)}_{\mathbf{u}_k} + \underbrace{\left(\begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha_v \end{array}\right)}_{\boldsymbol{\alpha}_k} \\ \mathbf{y}_k & = & \underbrace{\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)}_{\mathbf{C}_k}\mathbf{x}_k + \boldsymbol{\beta}_k \end{array}\right.$$

We can thus use a Kalman filter with

$$\hat{\mathbf{x}}_{0|-1} = (1\ 2\ 0)^{\mathrm{T}} \text{ and } \boldsymbol{\Gamma}_{0|-1} = 10^{10} \left(\begin{array}{ccc} 1 & 0 & 0 \\ \cdot\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right).$$

and

$$\boldsymbol{\Gamma}_{\boldsymbol{\alpha}_k} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \text{ and } \boldsymbol{\Gamma}_{\beta_k} = \left(\begin{array}{cc} 10^2 & 0 \\ 0 & 10^2 \end{array}\right).$$