Examen Inertiel, ENSI 2.
Luc Jaulin, ENSTA-Bretagne

Le vendredi 10 janvier 2020. Appareils électroniques interdits.
Polycopié et notes manuscrites autorisés
Durée: 60 minutes.

Exercise 1. Consider the matrix

$$\mathbf{R} = \begin{pmatrix} 1 & a & d \\ 0 & b & 1 \\ 0 & c & 0 \end{pmatrix}$$

1) Find a, b, c such that we have a rotation matrix.

2) Give a representation of the rotation using Euler angles. Give a representation of the rotation using quaternions.

3) Give the value of $\mathbf{R}^k$, for $k \in \{-1, 0, 1, 2, 3, 4, \dots\}$.

4) Give an expression of this matrix of the form $\mathbf{R} = \exp(\mathbf{A})$. Give the value for $\mathbf{A}$.

Exercise 2. We consider a wheel with center at the origin O. The axis of the wheel has the direction Ox. The angle of the wheel with respect to some reference point located on the tire of wheel is denoted by φ. No other rotation are possible. A torque u can be used to control the rotation speed ω_x of the wheel. The inertia matrix of the wheel is

$$\mathbf{I} = \begin{pmatrix} \frac{m\rho^2}{2} & 0 & 0 \\ 0 & \frac{m\rho^2}{4} & 0 \\ 0 & 0 & \frac{m\rho^2}{4} \end{pmatrix}$$

1) Find a state space model of the wheel.

2) Propose a controller so that the wheel converges to the constant desired speed $\bar{\omega}$.

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2) A controller could be $u = \bar{\omega}_x - \omega_x$.