Examen guidage, 2A Robotique

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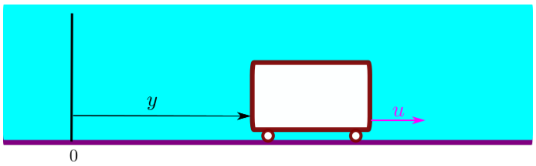
2023, April 04, 14h-15h30.

Appareils électroniques et polycopié interdits.

Notes manuscrites autorisées

Exercise 1. Consider a car moving horizontally as illustrated by the figure



Its evolution obeys the following differential equation $\ddot{y} + \dot{y}.|\dot{y}| = u$, where y is the position of the car and u is the force.

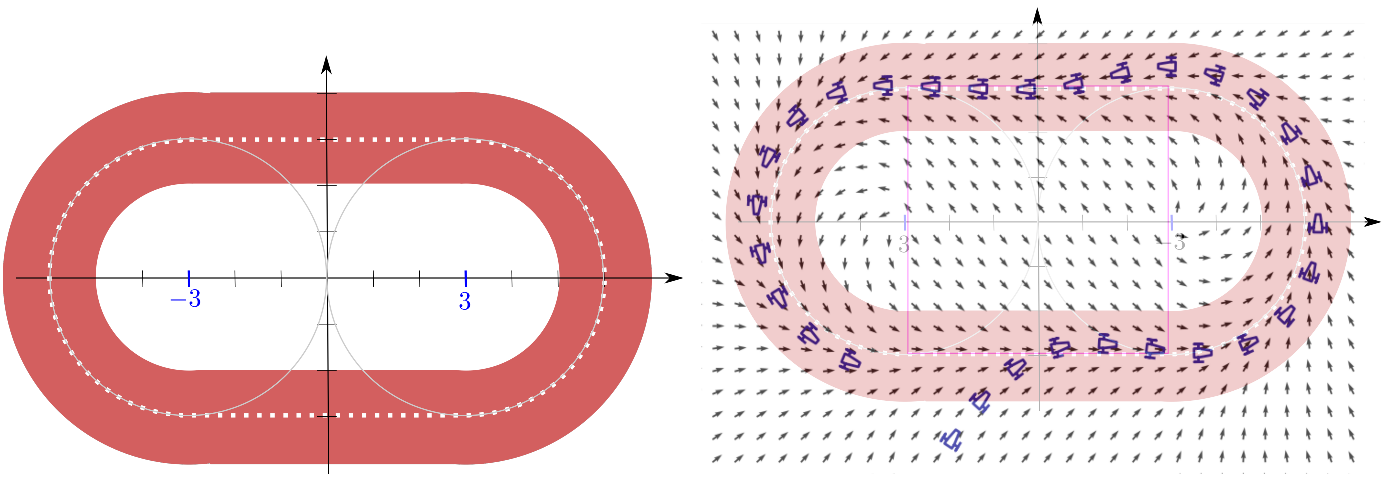
1) Give the state equation for this system. We will take as state vector $\mathbf{x} = (y \;\; \dot{y})^{\mathrm{T}}$.

2) Linearize around its equilibrium point. Study the stability of the system.

3) Using a feedback linearization method, propose a controller such that $y(t)$ converges to $y_d(t) = 3 \cdot t$.

Exercise 2. Consider a robot moving on a running track (see figure, Left) and described by:

$$\left\{ \begin{array}{lll} \dot{x}_1 & = & \cos x_3 \\ \dot{x}_2 & = & \sin x_3 \\ \dot{x}_3 & = & u \end{array} \right.$$



1) The expression of a vector field converging counterclockwise to a circle of radius 1 is given by:

$$\begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} -p_1^3 - p_1 p_2^2 + p_1 - p_2 \\ -p_2^3 - p_1^2 p_2 + p_1 + p_2 \end{pmatrix}$$

Find the expression of a counterclockwise vector field attracted by a circle of radius 3 and center $\mathbf{c}$.

2) Propose a controller so that the robot follows the running track as illustrated by the figure Right.

Correction, guidage, 2A Rob
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Exercise 1.

1) We have the following state equations:

$$\left\{\begin{array}{lcl}\dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -x_2\cdot|x_2|+u.\end{array}\right.$$

2) The linearized system is therefore written as:

$$\dot{\mathbf{x}}=\left(\begin{array}{cc}0 & 1 \\ 0 & 0\end{array}\right)\mathbf{x}+\left(\begin{array}{c}0 \\ 1\end{array}\right)u.$$

The two eigenvalues are nil, whence the instability of the linearized system. Let us note however that the liquid friction does not appear in the expression of the linearized system. This means that for an input $u=0$, the speed x_2 of the linearized system remains unchanged, whereas for the non-linear system, the speed x_2 converges towards 0.

3) We want

$$\begin{array}{ll} & \ddot{e}+2\dot{e}+e=0 \\ \Leftrightarrow & \ddot{y}-\ddot{y}_d+2(\dot{y}-\dot{y}_d)+y-y_d= \\ \Leftrightarrow & u-x_2.\,|x_2|+2(x_2-3)+x_1-3t=0\end{array}$$

The controller is this given by

$$u=x_2.\,|x_2|-2(x_2-3)-x_1+3t.$$

Exercise 2.

1) The vector field is given by

$$\boldsymbol{\varphi}_{\mathbf{c}}(\mathbf{p})=\mathbf{D}\cdot\boldsymbol{\varphi}_0(\mathbf{D}^{-1}\cdot(\mathbf{p}-\mathbf{c}))$$

where

$$\mathbf{D}=\left(\begin{array}{cc}3 & 0 \\ 0 & 3\end{array}\right)$$

and

$$\boldsymbol{\varphi}_0(\mathbf{p})=\left(\begin{array}{c}-p_1^3-p_1p_2^2+p_1-p_2 \\ -p_2^3-p_1^2p_2+p_1+p_2\end{array}\right)$$

2) We decompose the plane into 4 quadrants

$$\begin{array}{lccc} R & = & [3,\infty]\times\mathbb{R} & \text{(Right)} \\ L & = & [-\infty,-3]\times\mathbb{R} & \text{(Left)} \\ U & = & [-3,-3]\times[0,\infty] & \text{(Up)} \\ D & = & [-3,-3]\times[-\infty,0] & \text{(Down)}\end{array}$$

The field to be followed is given by

$$\boldsymbol{\varphi}(\mathbf{p})=\left\{\begin{array}{cl}\boldsymbol{\varphi}_{(-3,0)}(\mathbf{p}) & \text{if } \mathbf{p}\in L \\ \boldsymbol{\varphi}_{(3,0)}(\mathbf{p}) & \text{if } \mathbf{p}\in R \\ (-1,atan(-p_2+3)) & \text{if } \mathbf{p}\in U \\ (1,atan(-p_2+3)) & \text{if } \mathbf{p}\in D\end{array}\right.$$

The control low is

$$u=-\text{sawtooth}(x_3-\text{arctan2}(\boldsymbol{\varphi}))$$

See https://replit.com/@aulin/athletic