

## Examen localisation, ENSTA-Bretagne, ENSI 2.

Le 31 mars. A faire sur l'ordinateur.

Tous documents autorisés. Durée: 1h15.

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**Exercise 1.** We consider a boat the motion of which is described by the discretized state equation:

$$\begin{cases} x_1(k+1) &= x_1(k) + x_2(k) \cdot u(k) \\ x_2(k+1) &= x_2(k) + \alpha_2(k). \end{cases}$$

We measure the input  $u(k)$ . At the initial moment, we know that  $x_1$  is zero and that  $x_2$  is close to 1.  $x_2(0)$  will be represented by a Gaussian distribution whose mean is equal to 1 and whose standard deviation is 0.02. The scale factor  $x_2$  evolves slowly by means of  $\alpha_2(k)$  that we will assume to be centered, white and of standard deviation 0.01.

- 1) We apply an input  $u(k) = 1$  for  $k = 0, \dots, 9$  and  $u(k) = -1$  for  $k = 10, \dots, 19$ . Write a MATLAB program that implements a predictive Kalman filter capable of estimating  $x_1(k)$ .
- 2) Draw the confidence ellipses associated with the probability  $\eta = 0.99$ . How does the uncertainty evolve for  $x_1$  in function of  $k$  ?
- 3) In function of  $k$ , draw the determinant of the covariance matrix  $\mathbf{\Gamma}_x$ . Discuss.

You should send by email a pdf file containing all figures, the explanations and the Matlab code to [lucjaulin@gmail.com](mailto:lucjaulin@gmail.com).

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