Estimating the success probability of a set event in a probabilistic world

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Keywords: Probabilities, Interval Methods, Underwater Robotics

Introduction

Because of the constraints inherent to their environment, underwater robots carry out most of their missions autonomously. For critical applications, such as mine warfare, it is essential to be able to guarantee the success of a mission or, at least, to estimate its probability of success.

Missions are often achieved by validating a set event, such as the coverage of a specific area by a sensor, or the respect of a navigation corridor. However, sensor modeling and navigation algorithms are generally based on probabilistic assumptions, such as Gaussian noise and Kalman filtering. So, to estimate the success probability of such mission, it is necessary to find a method that conciliates the ensemblist approach with the probabilistic one, without confusing the two.

Formalism

Given a random trajectory $\mathbf{x}(\cdot) : \mathbb{R} \mapsto \mathbb{R}^n$, with a known probability density function $\pi_{\mathbf{x}}$, and a random vector $\mathbf{p} \in \mathbb{R}^m$, with an unknown probability density function $\pi_{\mathbf{p}}$ but a known support $[\mathbf{p}] \in \mathbb{IR}^m$.

We consider an event E, defined with a function $h : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$, such as:

$$E : \forall \mathbf{p} \in [\mathbf{p}], \exists t \in \mathbb{R}, h(\mathbf{x}(t), \mathbf{p}) \leq 0. \tag{1}$$

By defining the boolean variable y, such as $y = 1$ if E is verified and $y = 0$ otherwise, we have:

$$P_E = P(y = 1) \in \int_{\mathbf{x}(\cdot) \in \mathbb{X}} [\eta](\mathbf{x}(\cdot)) \cdot \pi_{\mathbf{x}}(\mathbf{x}(\cdot)) \cdot d\mathbf{x}(\cdot) \tag{2}$$

where:

$$[\eta](\mathbf{x}(\cdot)) = \begin{cases} [1] & \text{if } \forall \mathbf{p} \in [\mathbf{p}], \exists t \in \mathbb{R}, h(\mathbf{x}(t), \mathbf{p}) \leq 0 \\ [0] & \text{if } \forall \mathbf{p} \in [\mathbf{p}], \forall t \in \mathbb{R}, h(\mathbf{x}(t), \mathbf{p}) > 0 \\ [0, 1] & \text{otherwise} \end{cases} \tag{3}$$

As a result, we will obtain an interval $[P_E]$, containing the probability P_E of the event E.

To estimate this interval of probability, we propose to use a Monte-Carlo method, generating $N \in \mathbb{N}^*$ samples of the trajectory $\mathbf{x}(\cdot)$ and verifying for each one the value of $[\eta_i]$ with $i \in [1, N]$ and where:

$$[\eta_i] = [\eta](\mathbf{x}_i(\cdot)) \text{ where } \mathbf{x}_i(\cdot) \text{ is the } i\text{-th draw of } \mathbf{x}(\cdot)$$

It is thus possible to obtain an estimate of $[P_E]$, by calculating the average of the $[\eta_i]$:

$$[P_E] \approx \frac{1}{N} \sum_{i=1}^{N} [\eta_i] \tag{4}$$

The law of large numbers leads us to believe that this estimate will be all the more accurate as $N \rightarrow \infty$.

Simulation

In this scenario, we consider a mobile robot described by a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ and initialized to a given state $\mathbf{x_0}$. It aims to follow a reference trajectory in the plane (represented by a black line in the Figure 1).

To do so, the robot evolves in dead-reackoning, with only a compass to estimate its heading (with a white Gaussian noise). Using a controller and a Kalman filter, it can generate a command $\mathbf{u_d}$ from the estimate of its state $\hat{\mathbf{x}}$. The robot's trajectory $\mathbf{x}(\cdot)$ is therefore random, and $N \in \mathbb{N}^*$ draws from this trajectory can be obtained by simulating the system N times.

We now introduce into the scenario three beacons whose positions are uncertain. The position $\mathbf{p}_j \in \mathbb{R}^2$ of the j-th beacon is in the set $[\mathbf{p}_j] \in \mathbb{IR}^2$. To detect the beacon, the robot must be close enough to it, i.e. at a distance less than or equal to the robot's detection radius r_b. We can thus define a function $h : \mathbb{R}^2 \times \mathbb{R}^2 \mapsto \mathbb{R}$ to quantify this statement:

$$h(\mathbf{x}, \mathbf{p}) = \|\mathbf{x} - \mathbf{p}\| - r_b \tag{5}$$

The event E_j describes the case where the robot has seen the j-th beacon, and can be formalised by the following condition:

$$E_j : \forall \mathbf{p}_j \in [\mathbf{p}_j], \exists t \in \mathbb{R}, h(\mathbf{x}(t), \mathbf{p}_j) \leq 0 \tag{6}$$

For a given, and previously calculated, trajectory $\mathbf{x}_i(\cdot)$, with $i \in [1, N]$, it is therefore possible to estimate the probability of the event E_j by introducing the function $[\eta_j]$:

$$[\eta_j](\mathbf{x}_i(\cdot)) = \begin{cases} [1] & \text{if } \forall \mathbf{p}_j \in [\mathbf{p}_j], \exists t \in \mathbb{R}, h(\mathbf{x}_i(t), \mathbf{p}_j) \leq 0 \\ [0] & \text{if } \forall \mathbf{p}_j \in [\mathbf{p}_j], \forall t \in \mathbb{R}, h(\mathbf{x}_i(t), \mathbf{p}_j) > 0 \\ [0,1] & \text{otherwise} \end{cases} \tag{7}$$

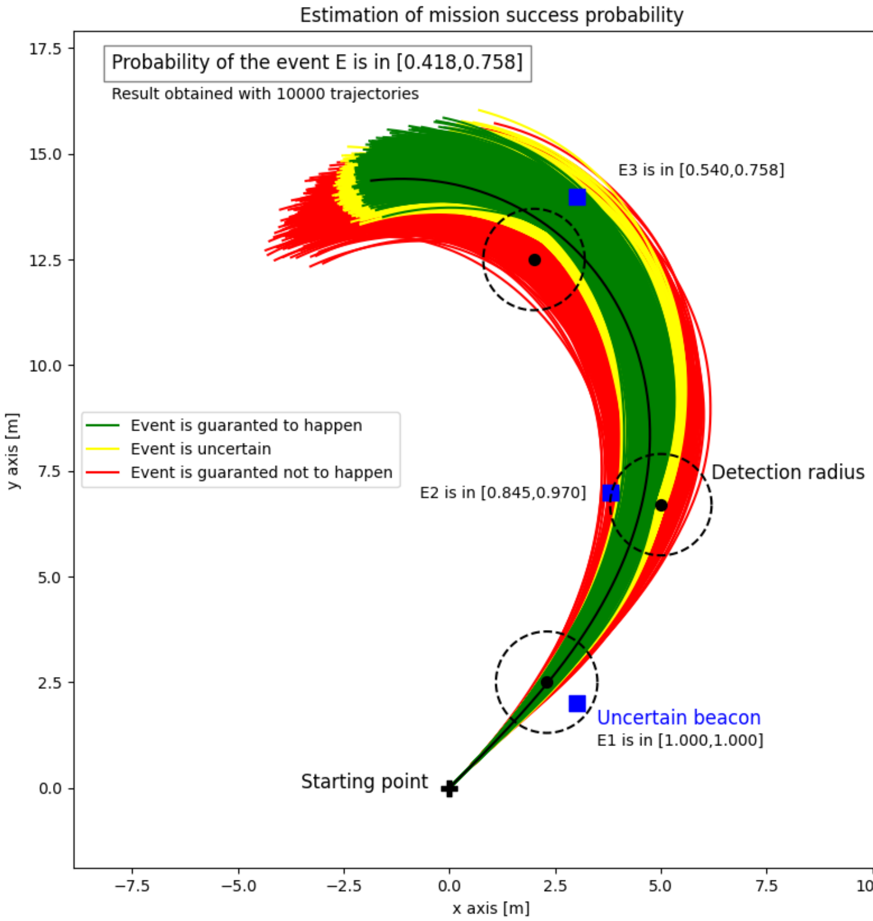

Figure 1: Simulation of the scenario

By calculating the value of $[\eta_j]$ for each trajectory $\mathbf{x}_i(\cdot)$ and then averaging, it is now possible to obtain an estimate of $[P_{E_j}]$.

In this scenario, it is even possible to go one step further and estimate the probability that the robot will detect all 3 beacons during a single trajectory $\mathbf{x}_i(\cdot)$. This mission is described by the event E, with $E = E_1 \wedge E_2 \wedge E_3$, and the function $[\eta_j]$ is replaced by the function $[\eta]$ where:

$$[\eta](\mathbf{x}_i(\cdot)) = [\eta_1](\mathbf{x}_i(\cdot)) \wedge [\eta_2](\mathbf{x}_i(\cdot)) \wedge [\eta_3](\mathbf{x}_i(\cdot)) \tag{8}$$

where $\wedge$ should be interpreted using the three valued logic.