

Introducing Box Chains to simplify Reachability Analysis

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Introduction

Reachability Analysis is an important tool in robotics when it comes to ensure that a robot performs its mission safely. Performing a Reachability Analysis comes down to calculating the image of a disk D by a function f . In this work we will assume that for all x in D we have $\det(J_f(x)) > 0$ where \det is the determinant function and J_f the Jacobian matrix of f . This means that if we choose an orientation for the contour ∂D of D , it is conserved by the function f .

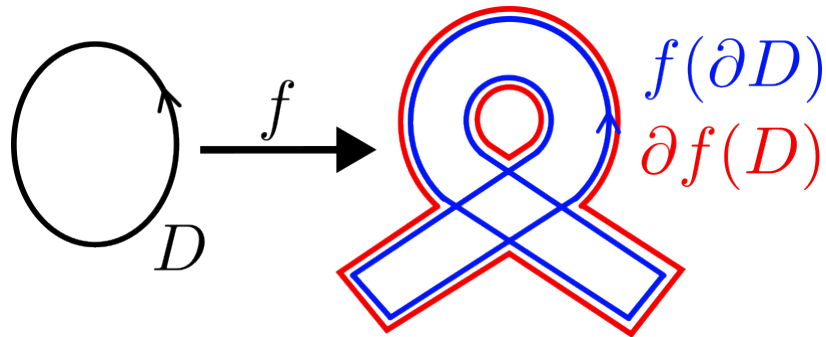


Figure 1: Reachability Analysis

Different methods [1] [2] allow us to estimate the reachable set of a robot at a given time. We will focus on the method presented in [3] which provides the contour of the Reachable set and its normal. We will see how we can efficiently apply it with Interval Analysis tools.

More specifically we will introduce the notion of "Box Chain" and see how it can be used to detect fake boundaries in two dimensions and greater. If we adopt the notations in fig 1 , eliminating fake boundaries comes down to finding $\partial f(D)$ from $f(\partial D)$.

Main results

We propose a definition of the Box Chain and use it to partition a contour in two dimensions. This partitioning allows us to detect intersections in the contour, meaning that there are fake boundaries in it.

The main contribution of this work is to show how we can use these Box Chains to detect fake boundaries. This method uses the fact that we know the normal to the contour to first color the inside of the Reachable set, and then suppress the fake boundaries.

We will finally see how this method can be extended in higher dimensions.

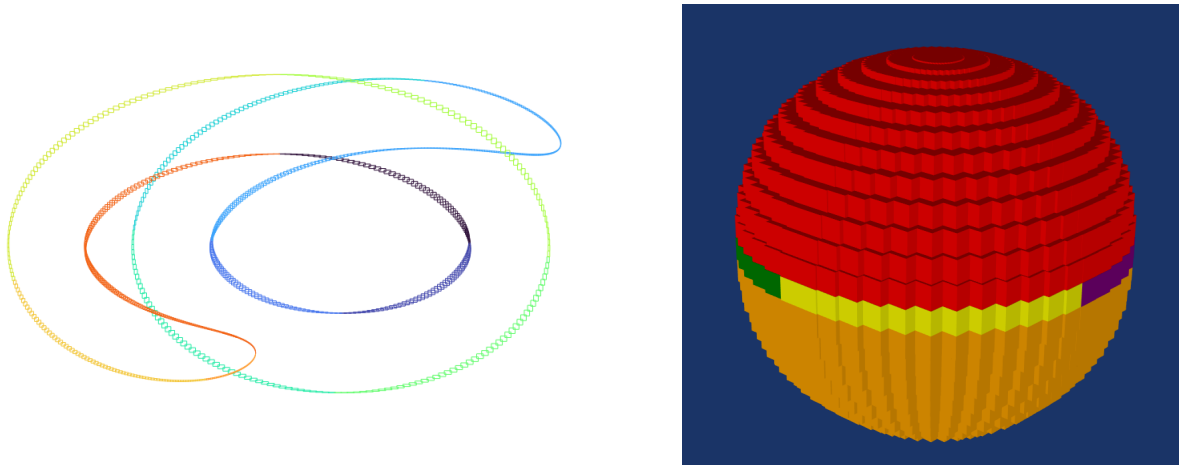


Figure 2: Box Chain decomposition of a 2D and a 3D contour

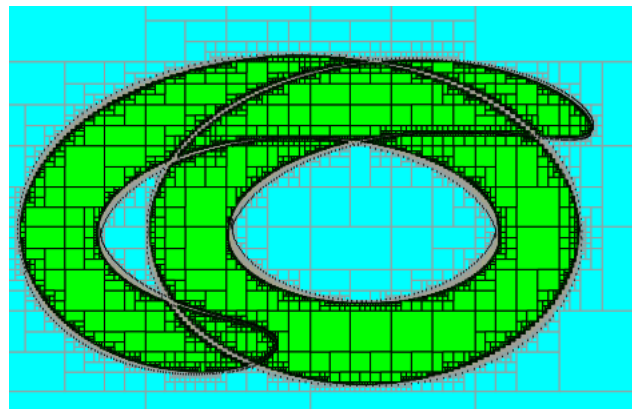


Figure 3: Inside of the Reachable set colored in green

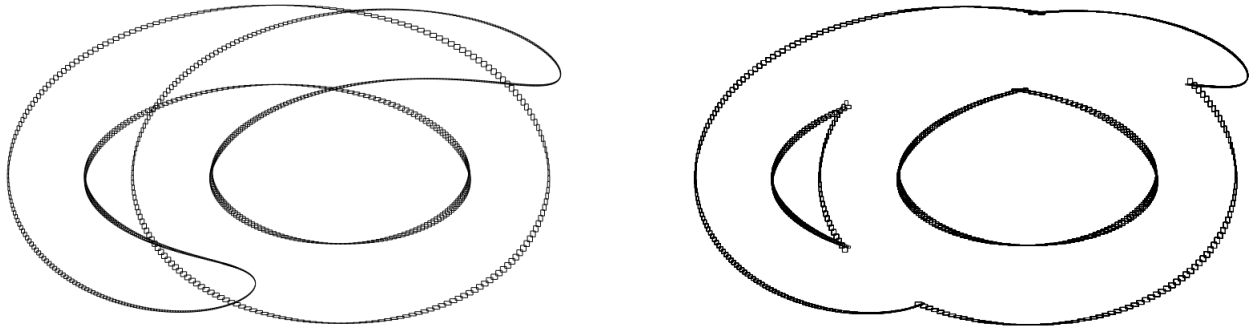


Figure 4: Eliminating fake boundaries

Acknowledgement

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References

- [1] J. Damers, L. Jaulin and S. Rohou. *Lie symmetries applied to interval integration*. Automatica 2022.
- [2] L. C. G. J. M. Habets, P. J. Collins, and J. H. van Schuppen. *Reachability and Control Synthesis for Piecewise-Affine Hybrid Systems on Simplices*. IEEE Transactions on Automatic Control, 2006.
- [3] T. Lew, R. Bonalli and M. Pavone. *Exact Characterization of the Convex Hulls of Reachable Sets*. 62nd IEEE Conference on Decision and Control (CDC 2023), Dec 2023.