Reinforced Set Projection Algorithm

Nuwan Herath Mudiyanselage[1], Luc Jaulin[1]
and Simon Rohou[1]

[1] ENSTA Bretagne, Lab-STICC, UMR CNRS 6285, Brest, France
2 rue François Verny, 29806 Brest Cedex 09, France
lucjaulin@gmail.com,{nuwan.herath,simon.rohou}@ensta-bretagne.fr

Keywords: Set description, Set projection, Contractors, Separators

Introduction

With interval arithmetic, solution sets to given problems can be described by operators called contractors and separators. First, we recall what they are.

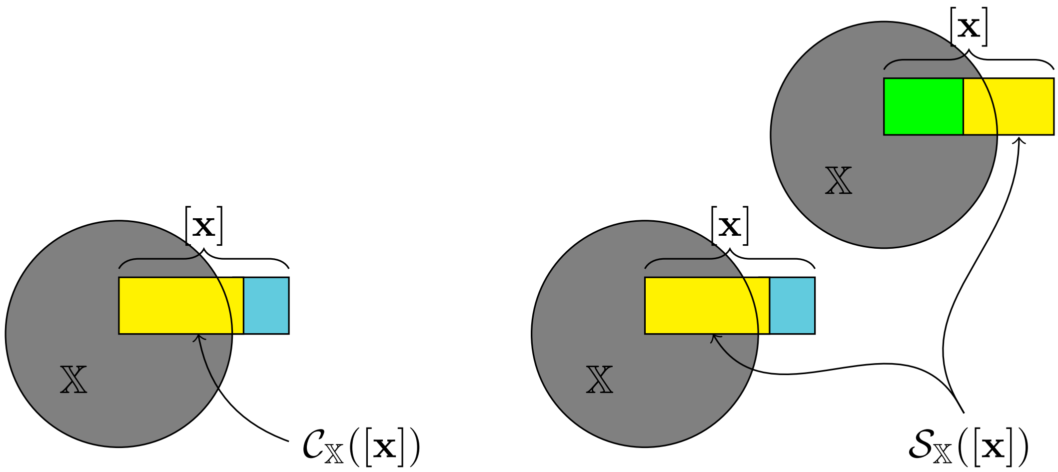
Set descriptors: from contractors to separators

Let us say that our solution set is $\mathbb{X}$. There exist interval operators to describe it [1]. $\mathcal{C}_{\mathbb{X}}$ denotes the contractor that describes the set $\mathbb{X}$. $\mathbb{IR}$ denotes the intervals of $\mathbb{R}$. One can apply the contractor on the box $[\mathbf{x}] \in \mathbb{IR}^n$ and get $\mathcal{C}_{\mathbb{X}}([\mathbf{x}])$, as illustrated by Figure 1a. $\mathcal{C}_{\mathbb{X}}$ verifies two properties:

$$\mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] \text{ (contractance) and } \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} \text{ (correctness).}$$

One can guarantee that $[\mathbf{x}] \setminus \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \not\subset \mathbb{X}$. It is then possible to construct a paving made of blue boxes that do not contain points from $\mathbb{X}$ and yellow boxes that may contain points from $\mathbb{X}$ (see Figure 2a). The latter form an exterior approximation denoted by $\mathbb{X}^+$.

Separators simultaneously provide inner and outer approximations of the set $\mathbb{X}$, as illustrated by Figure 1b. Thus, one can also identify green boxes that are contained in $\mathbb{X}$ (see Figure 2b). They form an



(a) Contractor (b) Separator

Figure 1: Contractor $\mathcal{C}_{\mathbb{X}}$ and separator $\mathcal{S}_{\mathbb{X}}$ applied on the box $[\mathbf{x}]$.

interior approximation denoted by $\mathbb{X}^-$. Green and yellow boxes form $\mathbb{X}^+$. We then have an enclosure: $\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+$.

Set projection separators

In some applications, one may only be interested in the projection of the solution set [2]. We have a separator for that operation: `SepProj` in the Codac library [3].

Reinforced projection separators

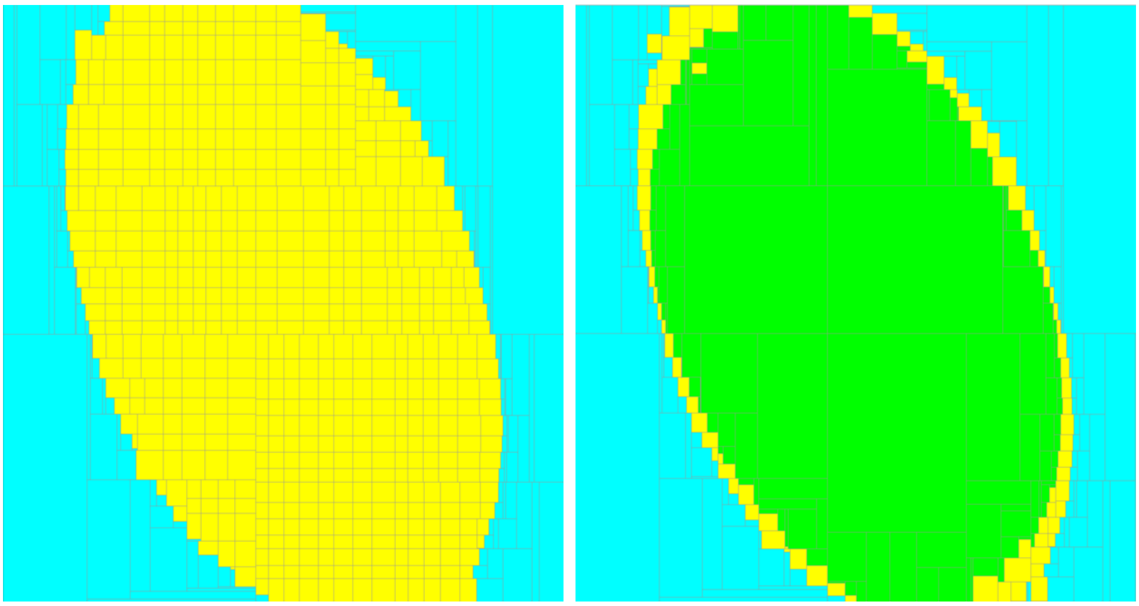
Let us look at the projection along the z-axis of the set $\mathbb{X}$ defined by

$$2x^2 + 2.2xy + xz + y^2 + z^2 \leq 10. \tag{1}$$

By defining $f(x, y, z) = 2x^2 + 2.2xy + xz + y^2 + z^2 - 10$, it can be written as

$$f(x, y, z) \leq 0. \tag{2}$$

The current implementation of the separator of the projection requires fine-tuning. Without the proper parameters, it produces bad quality



(a) Contractor $\to \mathbb{X}^+$ (b) Separator $\to \mathbb{X}^-, \mathbb{X}^+$

Figure 2: Pavings of the set $\mathbb{X} = \{x, y \in \mathbb{R}^2 \,|\, 2x^2 + xy + y^2 \leq 1\}$ for the two classes of descriptors.

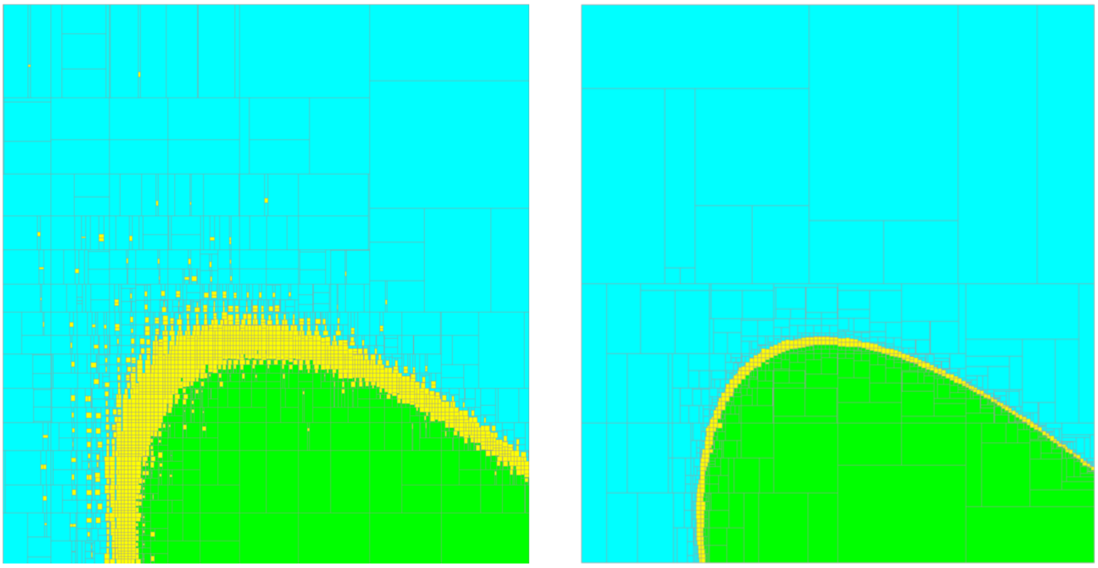
boundaries for our particular problem (see Figure 3a). The approximation is not minimal due to pessimistic results coming from interval dependency. Indeed, Equation 1 has multiple occurrences of the same variable. We present a new approach for differentiable sets which focuses on the boundary (see Figure 3b).

We reinforce the separator on $\partial \operatorname{Proj} \mathbb{X}$, the boundary of the projection. For our example, we use the knowledge of the locii of the vertical tangents to $\mathbb{X}$ which are defined by

$$\begin{cases} f(x, y, z) = 0, \\ \dfrac{\partial f}{\partial z}(x, y, z) = 0. \end{cases} \tag{3}$$

In our case, that is

$$\begin{cases} 2x^2 + 2.2xy + xz + y^2 + z^2 = 10, \\ x + 2z = 0. \end{cases} \tag{4}$$



(a) `SepProj` (b) Reinforced projection

Figure 3: Pavings of the projection along the z-axis of the set $\mathbb{X} = \{x, y, z \in \mathbb{R}^3 \mid 2x^2 + 2.2xy + xz + y^2 + z^2 \leq 10\}$.

`SepProj` was constructed from $\mathcal{S}_{\mathbb{X}}$ based on Equation 1. For the reinforced projection algorithm, we add $\mathcal{C}_{\partial\,\mathrm{Proj}\,\mathbb{X}}$ based on the knowledge of Equation 4.

References

[1] Gilles Chabert and Luc Jaulin. Contractor programming. *Artificial Intelligence*, Volume 173, Issue 11, 2009.

[2] Benoît Desrochers. Simultaneous Localization and Mapping in Unstructured Environments. Thesis, 2018.

[3] Simon Rohou, Benoît Desrochers and Fabrice Le Bars. The Codac Library. *Acta Cybernetica*, Mar. 2024.