Second Grade Internship Report

DYNAMIC PARAMETERS IDENTIFICATION

A BISECTION APPROACH BASED ON INTERVAL ANALYSIS

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Résumé

Le rapport se concentre sur les sujets des problèmes d'identification des paramètres sur un aéroglisseur. Une méthodologie basée sur l'analyse d'intervalle est donnée par l'auteur. Comme le travail de son stage, il s'est adonné sur la conception de l'algorithme dans le but d'identifier les paramètres avec les mesures indiquées. L'analyse par intervalles est utilisée pour estimer la borne de valeur réelle. Une méthode de Runge-Kutta est également fait usage de calculer tous les états. L'auteur utilise en même temps la méthode de bissection pour rechercher dans l'espace de solution. Compte tenu des inconvénients de la première version de l'algorithme, l'auteur a fait une optimisation sur l'algorithme. En conséquence, l'algorithme désignée optimisé bien résolu le problème. Les paramètres avec lesquels la simulation se fait tous se trouvent à la fin. L'auteur suggère que cet algorithme est bien marché pour un problème dynamique d'identification des paramètres. Considérant qu'il lance très lentement, c'est nécessaire d'optimiser l'algorithme pour une meilleure performance. Un algorithme assez rapide contribuera beaucoup à la technologie de l'industrie robotique.

Mots clés: Analyse intervalle; Identification de paramètres; Controle; Bisection

Abstract

This report focuses on the topics of a parameter identification problem build on a hovercraft. A methodology based on interval analysis is given by the author. As the job of his internship, he worked on the design of algorithm for the purpose of identifying the parameters with given measurements. Interval analysis is used to estimate the range of real value. A Runge-Kutta method is also made use of to calculate all the states. The author meanwhile utilize bisection method to search in the solution space. Considering the disadvantages of the first version of algorithm, the author did an optimization. As consequence, the optimized algorithm well solved the problem. The parameters with which the simulation is done are all found at last. The writer suggests that this algorithm is well-worked for a dynamic parameter identification problem. Whereas it launches very slowly. It is necessary to optimize the algorithm for faster performance. A fast enough algorithm will contribute a lot to the heated technology of advanced robot industry.

Keywords: Parameter Identification; Interval Analysis; Low of Control; Bisection

Contents

Chapter 1: Introduction

1.1 Organization of Internship

1.1.1 ENSTA Paristech

ENSTA ParisTech, or so-called École Nationale Supérieure de Techniques Avancées in French, is a teaching and public research institution that is self-governed under the supervision of the Ministry of Defence. It enjoys a special position in the French education system. It belongs to one of the most important schools of engineering in the country.

ENSTA ParisTech offers its students a general training in engineering in order to enable them to design, implement and manage complex technical projects, while meeting economic constraints in an international environment. In this perspective, the Institute offers training high scientific and technological level, which is constantly updated to keep pace with changes in advanced technology, and supplemented by the languages, humanities and skills in business life such as law, communication, economics, accounting and management.

Research is another main task of ENSTA. Its five departments conduct research in many areas, in partnership with French universities, European and international and public research organizations. Many CNRS research staff, INSERM and the École Polytechnique also worki on ENSTA's own professors sides of the institute. The departments carry out research mainly applied with industrial problems, but are also involved in the fundamental developments in the interest of scientific knowledge and aiming at technological breakthroughs. The research provides a dynamic contribution to the educational work of the school, behind its connection with scientific state-ot-the-art.

In December 2010, ENSTA Paristech and ENSTA Bretagne (former Ensieta), two engineering schools had created the complementary ENSTA group to enhance training and high-level research activities. This group combines two schools with their recognized performances in their areas of expertise: energy, transportation, marine engineering and large industrial systems. It offers two schools on development of an ambitious strategy of growth and international exposure for their engineering education and research activities. In a highly competitive environment for higher education, the height and visibility are key growth factors, and the creation of the ENSTA Group fits into this perspective.

1.1.2 Robotic Laboratory of ENSTA Paristech

The Robotics Laboratory, known more formally as Autonomous Systems and Robotics Team, do researches focused on mobile robot navigation, perception, vision board, motor learning and human-robot interaction. Their primary objective is the application of machine learning to real-world, such as service and assistive robotics, humanoid robotics, intelligent vehicles, and security.

The team was founded by Jean-Christophe Baillie, and as a result of his work on control architectures, he introduced the language of the interface URBI to control the robot. This language was also developed by GOSTAI, a spin-off company he created, now bought by Aldebaran Robotics.

The Robotics Laboratory is part of Computer Science and System Engineering Laboratory of ENSTA-Paristech in Paris, France. Several team members are also part of the team INRIA / ENSTA-Paristech FLOWERS, co-located in Palaiseau and Bordeaux, which focuses on developmental robotics.

1.2 Subject Background

The robotic laboratory of ENSTA Paristech has recently designed a remote controlled hovercraft(Figure 1.1), which is equipped with electronic components as well as an Arduino board for central control.

The construct of this drone like robot is based on dynamic model [1], which is an approximation of a real hovercraft. Even though it is a simplified model, it has almost all the most important characteristics of a hovercraft. It is necessary to build a method for identification of dynamic parameters (mass, inertia, friction coefficients, etc.) so that the model is as close to reality as possible [2].

In this area, we work extensively with interval analysis which allows us to use the range of value in place of a solid number.This is because of two reasons: on one side, in the real environment all the acquired data is with errors; on the other side, all component

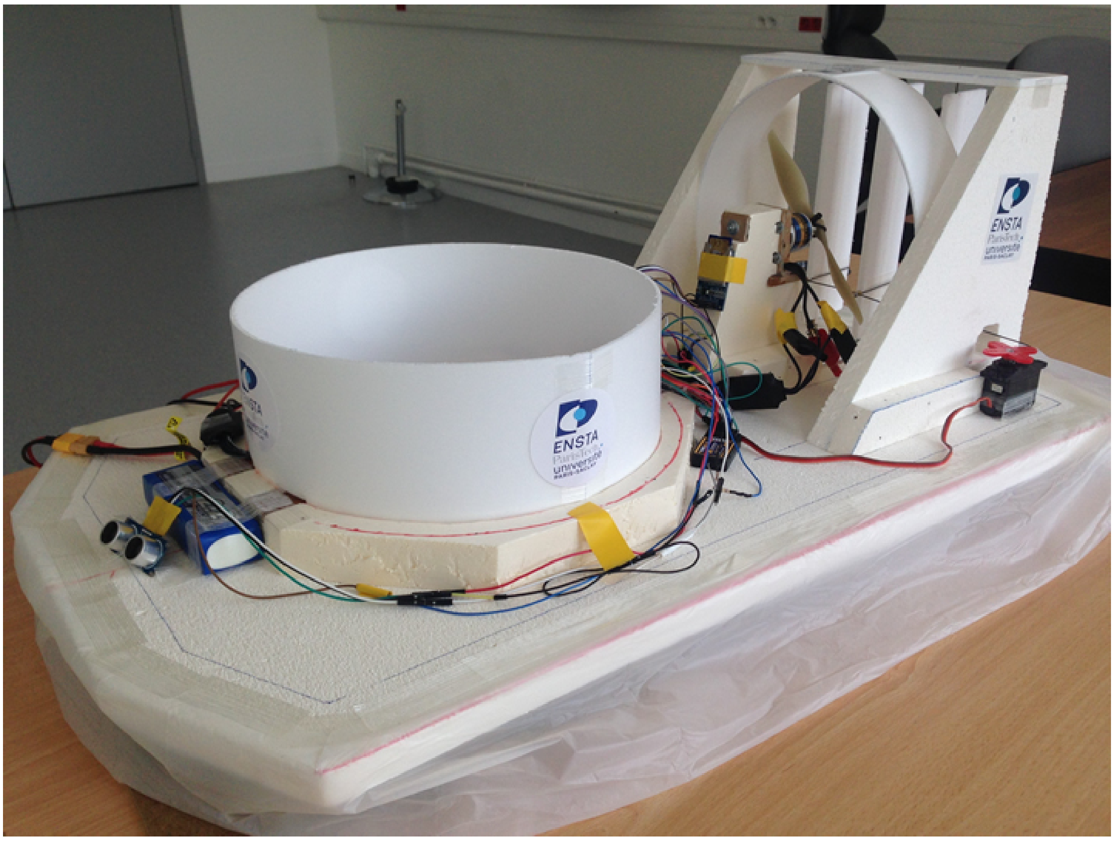


Figure 1.1: Simplified Model of a Hovercraft

have little motion when the hovercraft flies. Using this mathematical tool,all data are represented by a range, thus it is possible to make sure the calculation of all parameters and measurements well fits the reality.

Once such a model obtained and validated by the measurements, it will be interesting to define a law of control (or several), simply and robustly, for the hovercraft to accomplish a path under aimless situation (which means without sensor) or even more independent to pass an area with barriers without any prior knowledge of this area. This can also contribute a good way of the heated unmanned issue. In addition, with more than one path, it is easier to get more state data, which helps a lot to judge if the parameters are correct.

1.3 Missions to Realize

- Define the measurements to be performed for the identification of parameters;

  There are several measurements for the stare equations, in which some are very useful while some not, some are possible to measure and the others not. So the first thing is to choose the measurements to use.

- Choose the parameters to calculate by orders;

  The stare equations are represented with help of lots of parameters, but the coefficient of each parameter is rather different. In a mathematical way, all the parameter should be chose by an order of importance.

- Acquire the measurements (with Arduino program and experiments);

  This mission allows us to get data for calculation, but the data can be generated randomly as a simulation at the first time.

- Develop a tool for the identification of dynamic parameters;

  Write an algorithm or a program that identify the parameters.

- Design one or more control laws based on the simulation hovercraft.

  On control law is not enough, should more different path be designed. And repeat the steps on the new control laws helps us get more data, which can be used for correct the errors and optimize the parameters.

Chapter 2: Preparation Work

2.1 Theoretical Basis

2.1.1 Model-based Control and Parameters Identification

Model-based control is a kind of robot control method based on the theoretical physics model [1] [2] [3]. In the research of intelligent robots, a very important subject is how to correctly establish the mathematical model of the manipulated object, in order to construct a corresponding control [4].

The model is described by a series of parameters. Typically, they can be identified by static parameters identification methods [3] [5]:

1. Physical experiments, in which we measure the physics parameter directly.
2. Computer aided design (CAD) techniques, which means a graphic simulation by computer.
3. Black Box method, which use a mathematical tool to analyze the function between input and out put.

However, none of the static methods can adapt to the requirements of high accuracy in a real-time environment [6], While dynamic parameters can be used for describing the dynamic kinematics model of a robot, more precisely. They are important for advanced control algorithms based on models [7], which is because they have great influence on the validation of simulation results as well as the accuracy of algorithms designed for path planning. Nowadays, the requirements to accuracy and reliability of the system becomes increasingly strict, especially in the field of mechatronics robotics.

With all the parameters of a model identified, we can do all kinds of experiments by simulation on the computer. And if these parameters are accurate enough, there will be no difference between the results of simulation and actual experiments. Thus we can build a perfect control algorithm for the robots.

However, the ideal situation does not exist [8]. The parameters have uncertainties, and can be very sensitive to the errors. Therefore we utilize a mathematical tool which is called Interval Analysis [9].

2.1.2 Interval Analysis

In mathematics, interval analysis is a method for automated error estimation on the basis of closed intervals . In this case, x is not considered as an exactly known real variable, but a range which can be limited by two numbers a and b. X can lie between a and b or can also assume one of the two values. This range corresponds mathematically to the interval [a, b]. A function f, which depends on such an uncertain x, can not be evaluated exactly. Finally, it is not known that numerical value should actually be used within [a, b] for x. Instead, the smallest possible interval [c, d] is determined, which contains the possible function values f (x) for all x $\in$ [a, b]. By means of a targeted estimation of the end points c and d, a new function is obtained, which is a reflection from intervals to intervals [10].

This concept is suitable, among other things, for the treatment of rounding errors directly during the calculation and in case of uncertainties in the knowledge of the exact values of physical and technical parameters. The latter often result from measurement errors and component tolerances. Moreover, interval analysis can help to obtain reliable solutions of equations and optimization problems.

Figure 2.1 is an example of interval analysis application done on the platform of Ibex, a C++ library based on interval arithmetic for constraint processing over real numbers. The values at first not very accurate as represented in the blue range. But after a few steps calculation, the intervals shrink to yellow range and finally even in the black line at the center of the yellow range.

Interval analysis can also be used for error analysis in order to control the rounding errors resulting from each calculation [11]. The advantage of interval arithmetic is that after each operation, there is a gap which comprises reliably the true result. The distance between the ends of the range gives the current calculation rounding errors directly: Error = $\mathrm{abs}(a - b)$ to an interval [a, b]. Calculation with the two bounds is both clear and easy.

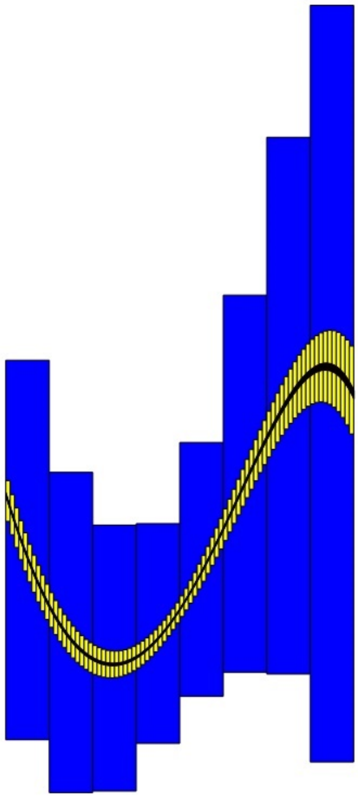


Figure 2.1: an Application of Interval Analysis

2.1.3 Runge–Kutta Methods and DynIbex

The Runge–Kutta methods are a family of implicit and explicit iterative methods [12], which includes the well-known routine called the Euler Methods, used in temporal discretization for the approximate solutions of ordinary differential equations.

DynIbex is a plug-in of Ibex library for constraint processing over real numbers. It offers a set of validated numerical integration methods based on Runge-Kutta schemes to solve initial value problem of ordinary differential equations and for DAE in Hessenberg index 1 form.

2.2 Mathematical Model of the Hovercraft

We work directly on the hovercraft model designed by robotic laboratory of ENSTA Paristech [1]. This is a reduced model, but with a complex behavior. The state equations can be given as follow:

$$\dot{u} = vr + \frac{1}{m}X \tag{2.1a}$$

$$\dot{v} = -ur + \frac{1}{m}Y \tag{2.1b}$$

$$\dot{r} = \frac{1}{I_z}N \tag{2.1c}$$

$$\dot{x} = \cos(\psi)u - \sin(\psi)v \tag{2.1d}$$

$$\dot{y} = \sin(\psi)u + \cos(\psi)v \tag{2.1e}$$

$$\dot{\psi} = r \tag{2.1f}$$

X, Y and N are denoted as the force and the moments in the whole system. They may come from the propulsion of motor, the air resistance , the friction on the ground or the damping of each rudder. X and Y are represented on the horizon plat, while N stands for the vertical direction. All the 3 variables are orthogonal. They are given by these equations:

$$X = F_u - \mu N f(u)\frac{u}{u^2+v^2} - \frac{1}{2}\rho C_d S_u \sqrt{u^2+v^2}u - D_{11}u \tag{2.2a}$$

$$Y = \frac{1}{4A}C_L S_v C_{F_u}\delta - \mu N f(v)\frac{v}{u^2+v^2} - \frac{1}{2}\rho C_d S_v \sqrt{u^2+v^2}v - D_{22}v \tag{2.2b}$$

$$N = -L\frac{1}{4A}C_L S_g C_{F_u}\delta - D_{33}r \tag{2.2c}$$

With constraints:

$$F_u = C_{F_u}W^2 \tag{2.3a}$$

$$C_{F_u} = C_\delta \rho D^4 \tag{2.3b}$$

$$A = \pi {R_h}^2 \tag{2.3c}$$

$$R_h = \frac{1}{2}D \tag{2.3d}$$

In the equations (1)-(3), we introduce many symbols, in which u means the velocity of hovercraft in x-axis direction, as well as v in y-axis direction. r stands for the angular velocity, which is useful when the hovercraft turns around. x, y and ψ are the position coordinate and angular direction corresponding respectively to u, v and r. f is a regularization function to avoid a discontinuity between the static and sliding friction. δ represent the law of control for path-finding. Except all these 8 variables, all the other symbols we use here are the parameters. Their physic meaning and units are shown in the following table:

Table 2.1: Parameters of Hovercraft Model.

Parameter	Physic Representation	Unit
m	mass	kg
D	propeller diameter	m
L	distance between gravity center and the rudder aerodynamic center	m
W	rotational speed	s^{-1}
I_z	moment of inertia according to z-axis	$kg \cdot m^2$
C_d	air resistance coefficient	$-$
C_L	rudder lift coefficient	rad^{-1}
C_{F_u}	propulsion coefficient	$-$
C_δ	dimensional propulsion coefficient	$-$
S_u	front area surface	m^2
S_v	side area surface	m^2
S_g	rudder area surface	m^2
A	propeller-rotation-generated surface	m^2
ρ	air density	$kg \cdot m^{-3}$
μ	friction coefficient	$-$
x_g	x-coordinate of the gravity center	m
y_g	y-coordinate of the gravity center	m
D_{11}	damping according to x-axis	$kg \cdot s^{-1}$
D_{22}	damping according to y-axis	$kg \cdot s^{-1}$
D_{33}	damping according to z-axis	$kg \cdot s^{-1}$

2.3 Parameter Selection

Fortunately, for such a huge group of equations, we have some common sense to help us determine the an approximate range of several parameters. For example, even if we do not know exactly the numerical value of m (mass), we can get it from a weighing scale. Consider that this hovercraft is rather light, our priory experience tell us this indication error will be less than 0.5 kg. We apply this technique to all the parameters, we can draw a word sketch of the whole system. Then we calculate the partial derivative of these equations, hence we get partial differential coefficients for all parameters, namely, the significance of each parameter to all the variables. Under this approach, we divide all the parameters into 3 categories: very important, negligible and the others (Table

2.2). Those parameters not appeared in the table has less contribution but cannot be totally ignore.

Table 2.2: Significance of Parameters.

Regarding to	Negligible	Important
$\dot{u}$	D_{11}, ρ, C_d, S_u	S_v
$\dot{v}$	D_{22}, ρ, C_d, S_v, A	
$\dot{r}$	D_{22}, I_z, A	L, S_g

To start, we use only the most important parameters S_v, L, S_g. These 3 parameters are set as intervals big enough for the purpose of finding the true value. And the others are set as a point value to simplify our first trail.

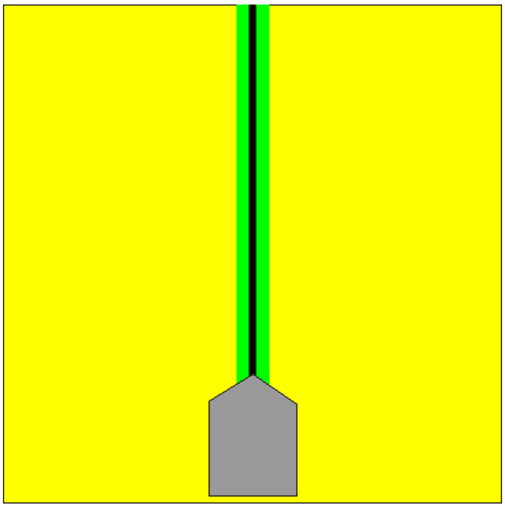
In this $\mathbb{R}^3$ space, all the dimensions are continuous. We configure a precision of 10^{-8} in case that the calculation never stops. That is to say, when the error is less than 10^{-8} it will be stopped. The initial values to launch the program are listed in the following form:

Table 2.3: Initial Values for Parameters.

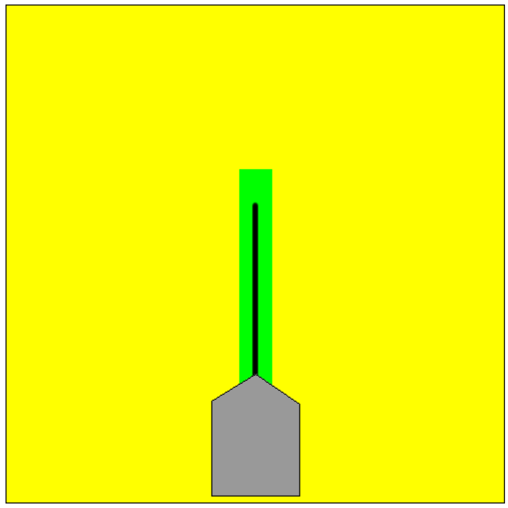
Point Values Parameter	
m	1.297
D	0.18
W	83.3
I_z	0.058
C_d	0.6
C_L	1.891
C_{F_u}	1.19576
C_δ	0.1339
S_u	0.1276
A	0.025447
ρ	1.225
μ	-0.5
x_g	0
y_g	0
D_{11}	0
D_{22}	0.2
D_{33}	0.1
Interval Values Parameter	
S_v	[0.20 , 0.25]
S_g	[0.0042 , 0.0754]
L	[0.25 , 0.35]

2.4 Designed Scenarios

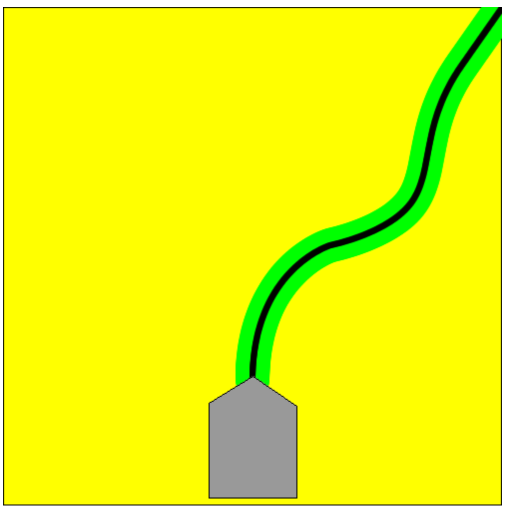
In real life environment, we may have different requirements for the robot. These requirements are also different path or different mode of fly of our hovercraft. More professionally, it is called a scenario. Here are the example of 4 different scenarios:



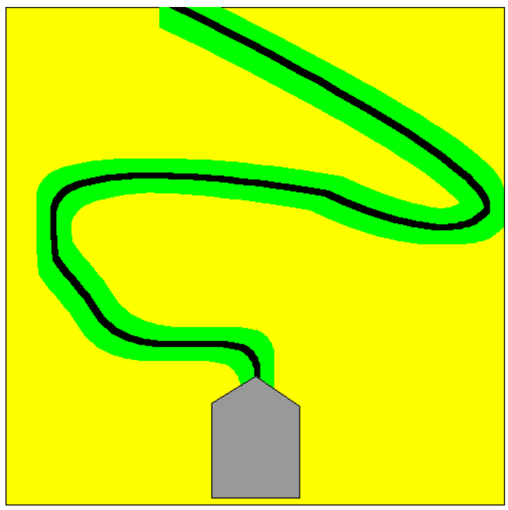
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 2.2: Example of Different Scenarios

We designed 3 scenarios:

1. The first one is a period of acceleration which allows to reach the stable speed $V_c = 2m/s$ in 1 second. The hovercraft follows a line in the direction of x-axis.

2. The second scenario is to let the hovercraft stop exactly in a distance between $3m$ and $7m$. It also follows a straight line.

3. The last design allows the hovercraft follow a curve. It should not stray away from the designed route so much. The rotation speed reduce to $\frac{1}{3}W_{V_{max}}$, where $W_{V_{max}} = 250$.

All the configurations of the 3 scenarios all given in the table followed:

Table 2.4: Different Designed Scenarios.

Items	Initial Condition	Rotate Speed	Control Law	Duration(s)
Symbol	$(u_0, v_0, r_0, x_0, y_0, \psi_0)$	$W(t)$	$\delta(t)$	t_f
Scenario 1	(0.1,0,0,0,0,0)	$W_{V_{max}}$	0	1
Scenario 2	(V_c,0,0,0,0,0)	0	0	10
Scenario 3	(V_c,0,0,0,0,0)	$\frac{1}{3}W_{V_{max}}$	$\frac{\pi}{12}(1 + \sin(\frac{\pi}{2}t))$	3

Chapter 3: Constructed Algorithms

3.1 First Algorithm

With the aim of conduct all the selected parameters in to a interval small enough, we make use of the method of interval analysis [9]. There is a library named Ibex which include thousands of useful functions of interval calculation. And through the instrumentality of DynIbex, a derivative tool from Ibex, we are able to solve initial value problem of ordinary differential equations under interval analysis method. It will be extremely helpful to our subject since the states equations are represented in the derivation form and all states should be calculated by calculus.

We also apply a so-called method of bisection, to divide the purpose interval into 2 parts each time. This allows us to search in all the resolution space.

At the beginning, all the measurements are generated randomly. Once the simulation is completely solve the problem, it should be applied on the real data. The algorithm is as given in Algorithm 1.

Algorithm 1 Simulation with the genarated measurements

Input: $Queue = \phi$, $Queue_{solution}$ = ϕ, $Queue_{uncertain}$ = ϕ,
The initial parameters [par]$_{init} \subset \mathbb{R}^n$,
The generated measurements M_k under the k-st scenario S_k
Output: $Queue_{solution}$

Add [par]$_{init}$ in $Queue$
while $Queue \neq \phi$ **do**
  Get a [par] from $Queue$
  For the scenario S_k, construct the state equations $\dot{y} = f_k(y, par)$
  Calculate the simulation $y_k(t)$ with parameters [par]
  if $\forall$ i, [$y_k(t_i)$] $\subset M(i)$ **then**
    Add [par] in $Queue_{solution}$
  else if $\exists$ i, [$y_k(t_i)$] $\cap\ M(i)$ = ϕ **then**
    continue
  else if $width([par]) > \epsilon$ **then**
    ([par]$_{left}$,[par]$_{right}$)=Bisect([par])
    Add [par]$_{right}$ in $Queue$
    Add [par]$_{left}$ in $Queue$
  else
    Add $[par]$ in $Queue_{uncertain}$
  end if
end while

3.2 Optimized Algorithm

Our first algorithm is feasible. At least it well solves the question to find the solution of parameters. Nevertheless, when sometimes a rather big random number appears (which stands for an accidental error or outliers in the experiment), the program refuse the same solution strictly, which may not be considered as a part of our original intention. Therefor, the algorithm is optimized so as to tolerate a few random error without sacrificing its accuracy.

It is with facility to have the idea in mind that we use a statistical manner to screen out the outliers. For the date who is so far away from their local average or global average, it is surely an error. Still we it always has some date influenced the statistical data of the whole group and made it on failure. And with experience from this failure, we have hereby our second version of optimized algorithm:

Algorithm 2 Simulation with the genarated measurements

Input: $Queue = \phi$, $Queue_{solution} = \phi$, $Queue_{uncertain} = \phi$,
The initial parameters [par]$_{init} \subset \mathbb{R}^n$,
The generated measurements M_k under the k-st scenario S_k,
The threshold for rate of outliers $threshold$
The dimension of the measurements dim

Output: $Queue_{solution}$

Add [par]$_{init}$ in $Queue$
while $Queue \neq \phi$ **do**
  Get a [par] from $Queue$
  For the scenario S_k, construct the state equations $\dot{y} = f_k(y, par)$
  Calculate the simulation $y_k(t)$ with parameters [par]
  if count(i,[$y_k(t_i)$] $\subset M(i)$)$>$ $dim \times (1 - threshold)$ **then**
    Add [par] in $Queue_{solution}$
  else if count(i,[$y_k(t_i)$]$\cap$ $M(i) = \phi$)$\geq$ $dim \times threshold$ **then**
    continue
  else if $width([par]) > \epsilon$ **then**
    ([par]$_{left}$,[par]$_{right}$)=Bisect([par])
    Add [par]$_{right}$ in $Queue$
    Add [par]$_{left}$ in $Queue$
  else
    Add $[par]$ in $Queue_{uncertain}$
  end if
end while

Observing the state data of progression, we noticed that it is rare for an error to occur. From another standpoint, the data fit the measurements either very well or very badly, there barely exist a situation that the result is refused due to the last few values [13].

For this reason, we add a counter to each sub-program so that we can count the number of faults that makes it refused or accepted. Then we set a threshold for error tolerance. Taking 10% as example, if all the other 90% values passed the verification, the whole result will be accepted.

3.3 Simulation Data Generation

To be on the safe side, our program does not begin with a real experimental environment. All the experiments were firstly done by simulation. Hence the first step is to generate the measurements for simulating. The procedure is as follows:

1. Define all the parameters p_{init} as constants with apriori experience. For the parameters to solve, make sure the value is chosen in its interval.
2. With all the parameter defined, calculate all the states for a duration t_f.
3. For each state, get its real corresponding measurements $[M_{real}]$.
4. Calculate the average M_{mid} of the lower bound and upper bound of M_{real}
5. Generate a random error ϵ in relation to the scale of M_{real}.
6. Define a rate of interval width s.
7. Get the final generated measurements $M_k = (M_{mid} + \epsilon) \cdot [1 - s, 1 + s]$

As we can see, M_k is also the input of our algorithm.

3.4 Real Experiment

All the word are done under simulation. The part of real experiment was not accomplished by me, thus will not be introduced in this report.

Chapter 4: Results and Conclusion

4.1 Results

The aim of algorithm validation is to confirm that it will perfectly suit the hovercraft model and has long term application. Thereby our crucial test is on the validity. We firstly define all the parameter and generate the measurements, then we launch the program with the parameters hidden and tried to find them back.

We apply this methodology to the hovercraft model, and it at last carried out the results. It find out the initial parameters successfully.

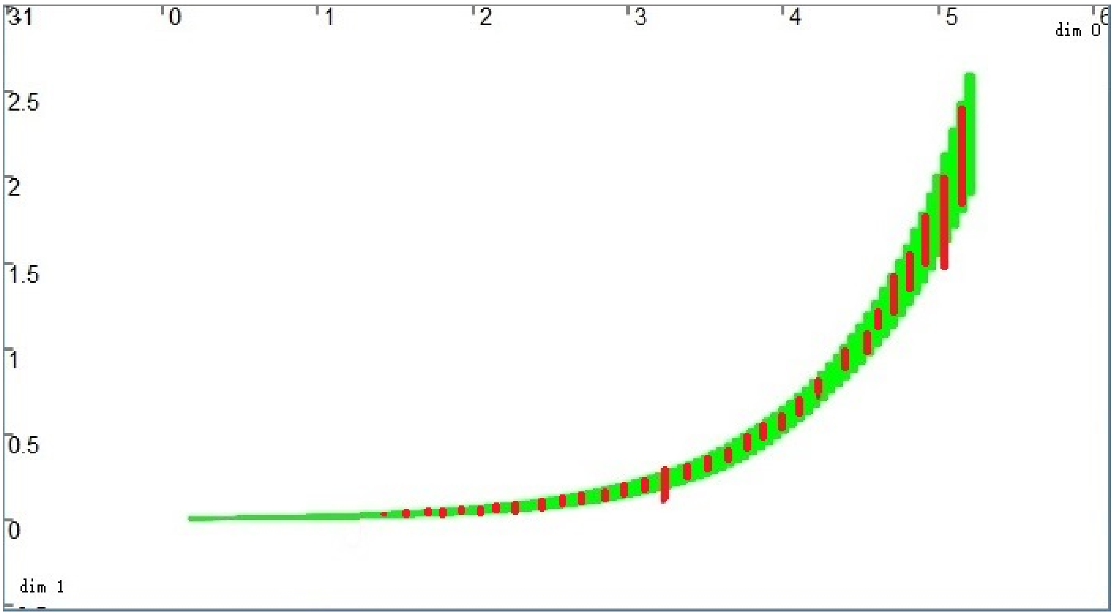


Figure 4.1: Mesuremensts well Fit the Intervals.

Here is an example shown in Figure 4.1. In order to show it more clearly and visibly, we extract one dimension of the states in a duration of 30 seconds rather than present all of them. We can see that there are 2 outliers in the measurements, which are added by purpose. These outliers are so enormous that cannot be ignored at all. This may be caused by static electricity on electric components or a wired wind. With such a group of data, the first algorithm fails while the optimized version accept with success in all the 3 different scenarios.

Given a error tolerance of 3 outliers, the program shows that there exists indeed a solution to our former problem. By the result, we can also analyze the possibility of our designed low of control [7].

4.2 Conclusion

In this report, we constructed a mathematical model based on an existent hovercraft. At the very beginning, all the theoretical basic concerned are reviewed. Next, based on the model of hovercraft, we analyzed the parameters and designed different scenarios. Furthermore, we constructed the algorithm to solve the core problem and let it be optimized. At last we did some simulation experiments and verified the algorithm. The result of this experiment is quite useful in further researches.

4.3 Acquisition

In the three months of internships, first of all, the most direct acquisition is on the professional techniques. I did not only practiced the knowledge I got from class, such as interval analysis, but also make me more skilled by learning a number of new expertise.

Meanwhile, working three months in a more professional environment, makes me better and faster adapt to working life and rhythm after graduation .

In the period of internship, I understood more about the needs of company and of our own development. Before, when someone asked me I would like to engage in what kind of job in the future, I can not answer it precisely, or can only answered that I want to word on IT. Now I get more recognized about myself.

4.4 Future Application

It is well-known that an accurate model is very important to improve the robot control [14], thus a precise parameter identification contributes a lot to the development of robotic science. As long as we get a model with very accurate parameters, we can do all kinds of control without any difficulty. Therefore, it will be useful to such kind of heated subject, like unmanned drive, self cruising or intelligent search.

Simultaneously, a series work of optimization should be done. Even if this algorithm can solve our problem, it is too long for a dynamic environment [15]. Imagined what happens when the drone meet a barrier and cannot get the result in 0.1 seconds. A good optimization of time complexity will do good to the expand of model-based control, as well as the prospect of the whole robot industry.

Chapter 5: Acknowledgment

There is no way to deny that it is a tremendous challenge for me to work in a real French environment. But thanks to some people's tireless efforts, I finished my internship as well as this report. Without their help, I may fail thousands of times and never had it done.

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Bibliography

[1] Julien Alexandre Dit Sandretto, Douglas Piccani de Souza, and Alexandre Chapoutot. Appropriate design guided by simulation: An hovercraft application. 2016.

[2] Julien Alexandre Dit Sandretto. *Étalonnage des robots à câbles: identification et qualification*. PhD thesis, Université Nice Sophia Antipolis, 2013.

[3] Jun Wu, Jinsong Wang, and Zheng You. An overview of dynamic parameter identification of robots. *Robotics and computer-integrated manufacturing*, 26(5):414–419, 2010.

[4] Wisama Khalil, Etienne Dombre, and ML Nagurka. Modeling, identification and control of robots. *Applied Mechanics Reviews*, 56:37, 2003.

[5] Vassilios D Tourassis and Charles P Neuman. The inertial characteristics of dynamic robot models. *Mechanism and machine theory*, 20(1):41–52, 1985.

[6] Sylvain Guegan, Wisama Khalil, and Philippe Lemoine. Identification of the dynamic parameters of the orthoglide. In *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on*, volume 3, pages 3272–3277. IEEE, 2003.

[7] Luc Jaulin. *La robotique mobile*. ISTE, 2015.

[8] Pradeep K Khosla and Takeo Kanade. Parameter identification of robot dynamics. In *Decision and Control, 1985 24th IEEE Conference on*, pages 1754–1760. IEEE, 1985.

[9] Hou Yangyi and Fang Hairong. Application of interval analysis theory in robot dynamics parameter identification. *Mechanics*, 35(4):64–66, 2008.

[10] Luc Jaulin. *Applied interval analysis: with examples in parameter and state estimation, robust control and robotics*, volume 1. Springer Science & Business Media, 2001.

[11] Matthew D Stuber and Paul I Barton. Robust simulation and design using parametric interval methods. In *4th International Workshop on Reliable Engineering Computing (REC 2010)*, pages 536–553. Citeseer, 2010.

[12] J Alexandre dit Sandretto and Alexandre Chapoutot. Validated explicit and implicit runge-kutta methods. *Reliable Computing*, 22:79, 2016.

[13] Andrès Vivas, Philippe Poignet, Frédéric Marquet, François Pierrot, and Maxime Gautier. Experimental dynamic identification of a fully parallel robot. In *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on*, volume 3, pages 3278–3283. IEEE, 2003.

[14] Stephen M Batill, John E Renaud, and Xiaoyu Gu. Modeling and simulation uncertainty in multidisciplinary design optimization. *AIAA paper*, 4803, 2000.

[15] R Serban and JS Freeman. Identification and identifiability of unknown parameters in multibody dynamic systems. *Multibody System Dynamics*, 5(4):335–350, 2001.