Internship Report

Camilo Ortiz
camilo.ortiz@ensta-bretagne.org

Résumé

In this report, I will detail my stay at INDESS, an institute of the university of Cadiz in Spain. I will talk about my role during the internship, how little by little my main objectives became clearer for me, how I dealt with some issues and how I approached my different tasks. The internship, as challenging as it was, came with its share of issues and questions. We will see in this report how I think it will help in my growth as an engineer. We will also talk about interval analysis and fuzzy numbers, as these were the main subjects of my internship. I will provide a few snippets of my code. Finally, we will talk about research and plagiarism, which is one of the most import subject that raised my attention during the internship.
Keywords : Interval, Fuzzy number, Research, Robot localisation, landmark, plagiarism

Dans ce rapport de stage, je vais détailler mon séjour à l'INDESS, un institut de l'université de Cadiz en Espagne. Je vais parler de mon rôle durant le stage, comment mes objectifs ont vu le jour petit à petit, comment j'ai été confronté à des problèmes et quelles méthodes m'ont permis d'approcher mes différentes tâches. Aussi stimulant soit-il, mon stage a eu son lot de problèmes et de questions dont la réponse n'est pas évidente. Nous allons voir dans ce rapport mon point de vue sur ma propre évolution en tant qu'ingénieur. Nous allons aussi évidemment parler d'analyse d'intervales et de nombres flous, étant donné que ces deux sujets sont les piliers de mon travail de recherche. Je vais donner des morceaux du code que j'ai écrit. Enfin, nous allons parler de recherche et de plagiat, car ce sont les sujets qui m'ont le plus fait réfléchir pendant la durée de ce stage et pendant l'écriture de mon article.
Mots clé : Interval, Nombre flou, Recherche, Localisation de robot, point de repère, plagiat

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1 Introduction

During the summer of 2023, between the 27^{th} of may and the 15^{th} of September, I had the opportunity to do an internship at INDESS, in the university of Cadiz in Spain. This research internship, in addition to helping me validate my studies in ENSTA Bretagne, made me discover mainly two things : What it is like to dedicate oneself to research, and also of course the beautiful region of Andalusia in Spain. My main objectives during this internship were to learn about research, and help me in my decision of doing a thesis after my cursus at ENSTA Bretagne. I also had the intention to learn more about robotic localization, and the utilization of the CODAC library as well as Interval Analysis.

2 Presentation of the institute

2.1 About Cadiz University

Cadiz University was born the 30^{th} of October, 1979, as the continuity of a long process of demand for a university institution that would follow the tradition of the province in regards to the development of maritime and commercial activities in modern and contemporary times. Nowadays, the university has developed and thrived, it currently has 4 campuses. The one I went to is the campus of Jerez de la Frontera, a city at the north of Cadiz, at a two hours drive from Seville. The campus was quite big with many facilities. A lot of buildings were used for the student's courses, and there were also buildings dedicated to some research groups. For instance INDESS had its own building. There is a well equipped sports facility, with a swimming pool and a gym that I had the opportunity to use.

Figure 1 – Logo of UCA

2.2 About INDESS

UCA contains various research teams and laboratories/institutes. The one that welcomed me is called INDESS (Instituto Universitario de Investigacion para el Desarrollo Social Sostenible). It was founded as a response to social and environment needs. Currently 147 researchers and scientists joined their respective knowledge for this institute. Its particularity is the diversity of scientific fields that are covered by the researchers in this institute. This results in INDESS being very versatile. Its main objective is to develop the whole university's knowledge and to transfer it to society. To manage that, INDESS chose sustainable development, health, economy

and criminology as its main axes of research. Those topics are represented jointly in the logo of the institute :

FIGURE 2 – Logo of INDESS

FIGURE 3 – Meaning of the logo

In the facility, I was assigned a room to work in. I shared this room with PhD students that I had the occasion to meet and discuss with. But we didn't work on the same subjects at all, which is why I didn't spend much time with them.

2.3 About my supervisor

My internship supervisor was Mr Arana-Jiménez. He is the director of INDESS and a mathematician with specialties in statistics and optimization problems.

3 My role during the internship

3.1 Setting my objectives

It is during the internship that my objectives came into existence. I found this internship thanks to Mr. Jaulin sending me the contact info of Mr Arana-Jiménez. When I started, Professor Jaulin just sent me an article to read about an application of fuzzy intervals to robotic localization. So if I knew that I would work with Interval analysis (which we had an introduction course in second year), I had no vision on what my internship might lead to. Little by little, by talking with my supervisor Professor Arana-Jiménez and with Professor Jaulin, we converged

towards one final task : make a new proposition of robot modelisation using fuzzy intervals, and write a science paper about it.

3.2 The documentation phase

The internship was split in different tasks. First I had to read a few papers about robot localization (and understand them!), about fuzzy intervals and all the algebra properties that were elaborated around those. This phase was quite important because it built my global knowledge of the state of the art. Most of those papers were either well known by my supervisors, or even written by them. So whenever I had questions about something that I didn't understand, I could ask my supervisors, thus I wouldn't get stuck on any mathematical aspect. But even with their help, understanding and assimilating all of those mathematical notions was a real hardship for me, mainly because I am not particularly good at math, but also because the topics were very advanced. Even though I read all those articles at the beginning of my internship, I often had to come back to some of them to make sure I understood well some notions, or to have a deeper understanding of those notions. There were a lot of articles to read, so I had to make a choice regarding what I consider relevant for my internship, and what is either too complex or to unrelated.

3.3 Back to the basics of Interval Analysis

Then I decided on my own to take some time to revise my lessons about interval analysis, and to further develop my basics about this subject. This was made easy thanks to my teacher Simon Rohou's lessons, and thanks to Mr Jaulin's online course and set of exercises. Those exercises are very useful because they go from beginner to very advanced. This took a few days but I don't regret it at all because it allowed me to better understand what I was doing in the rest of my internship. In parallel to that, my supervisor asked me to make him a small presentation around interval analysis (because this was not his domain of expertise). Thanks to that he could better guide me on what to work on during my stay at INDESS. Doing this presentation made me realize how efficient this institute was in its way of working with various different fields of expertise. Mr Arana-Jiménez managed to understand the technical aspects of interval analysis, even though I'm not a specialist in this domain and my presentation was probably not very precise, as I was learning about the subject myself. And after hearing my presentation, he managed to connect it with his field of expertise (fuzzy intervals) and come up with ideas of how to combine those topics. I believe this is one of INDESS's strengths, every researcher in the institute is very open to other ways of thinking and other forms of science, which is how they can easily put in common knowledge and make progress together.

3.4 Programming the KKT conditions

After that, in agreement with Professor Jaulin, my supervisor asked me to write in C++ a program inspired by the work of Mr Jaulin that would integrate to the problem the mathematical conditions of Karush-Kuhn Tucker. I spent a fair amount of time in that task because understanding those conditions and how they were integrated in the problem was very difficult, and writing it in C++ also ended up being very time consuming. Managing to do that would make me gain a lot of computing time in the future because these KKT conditions [6] allow to compute a contractor much more effectively. The most difficult part was debugging the program. It taught me a lot of things about C++ and the different compilation errors that can be raised. And because the article in which Mr Jaulin introduced this constraint is quite advanced, I got to learn about separators [5], which are an union of two contractors, an inside contractor and an outside contractor. Once I had finished with this task, Mr Jaulin asked me to implement the same algorithm but for a number n of landmarks and a number m of robots, given as parameters

to the program. This was not the hardest part of the internship. In order to achieve this, I just had to copy my previous code and use it in a loop.

3.5 Brainstorming the application of fuzzy intervals

Later on, we discussed with my supervisor on how to use fuzzy intervals in the modelisation of a robot, in a relevant way. First we had to choose what part of the algebra of fuzzy intervals was relevant for our problem. For instance we did not find interesting to mention the addition operation between two fuzzy intervals. This operation is not that trivial and would require some time to be integrated properly in my programs. Finally we decided to use those fuzzy intervals in the model of the measurements taken by the robot's sensors. We supposed that in a robot, there can be multiple sensors that will have the same function in the localization process. And instead of choosing between the two (or more) sensors, we can combine their approximation into a fuzzy interval. The rest of my internship was dedicated to working on that new idea. I spent some time on thinking about how to achieve it, then programming it, and then I worked on an article, as a way of displaying my work and results, on my supervisor's demand.

3.6 the redaction of an article

This last phase of my internship was about writing a science paper, with the help of Mr Jaulin and Mr Arana-Jiménez. And if it is good enough, maybe it will be published. This opportunity I was given, even though it was kind of optional because my internship had already ended, I knew I had to do it. Firstly for the experience it would provide me, but also for a potential line in my CV. That's why I considered it as a challenge. Currently, I have finished to write a draft and I am reviewing it with my supervisors. It took me a lot of time to achieve because I made the mistake of waiting too long before starting the redaction. Consequently, I had to dive again into the subject and into my programs, which was not easy due to the complexity of the topic. As it was the first time I faced the exercise of writing a science paper, I didn't know the standards for writing such a paper. For instance, I didn't know how much I could cite from my bibliography without it being considered as plagiarism. It's normal to reuse things like definitions, theorems or reasoning, particularly if it serves the interest of the article, but I still had to provide my own reflections and results, and not rely too much on what I had read. Another challenge during the redaction was to choose among all the results I had produced. Indeed, I did not just program one algorithm that I improved all along the internship. I created a lot of different scripts to try different ideas and concepts. I will put the most relevant code in the annexes.

4 Technical aspects of my internship

4.1 State of the art

When working on a mission, The robot needs to be able to make an estimation of its own position at any time in order to successfully realize said mission. In the field of Interval Analysis, we already have various methods that I learned in my courses with Simon Rohou. For instance [9] with the help of beacons, it is possible to have an estimation of the distance between the robot and each beacon. This distance equation, applied with 3 or more different beacons creates a constraint of overlapping circle areas. Basically, this is the way a system of geolocalisation (GNSS) works. From a computational point of view, the library CODAC [8] makes easy the overlap of those constraints : we simply have to create a contraction network and add as many constraints as we have landmarks. The constraint in this case is :

$$\mathbf{Z_j} = \{x \in \mathbf{R^2} | \sqrt{(x_1 - m_1(j))^2 + (x_2 - m_2(j))^2} \in [d_j]\} \tag{1}$$

where $\mathbf{Z_j}$ is the granule of possible positions for a robot of position x that we want to localize. The equation gives a ring shaped figure with the landmark of known position $m(j)$ at its center. d_j is the measurement of the distance made between the landmark and the robot. Simply put, a constraint is an equation which, written with intervals instead of float numbers, gives an area of possible solutions x. That is why we say we constraint the unknown value : we allow it to exist in a close bounded interval. Once the contraction network is set up, we can use a set inversion algorithm such as SIVIA (which we studied in second year at ENSTA Bretagne) [7] and contract the result in order to obtain an estimation of the robot's localisation. To display the result, we use alpha cuts. This topic will be detailed later on in the report. With alpha cuts, it is possible to display on the same picture the positions that validate at least two constraints, (that is the first alpha cut), then with another color the points that validate 3 and so on.

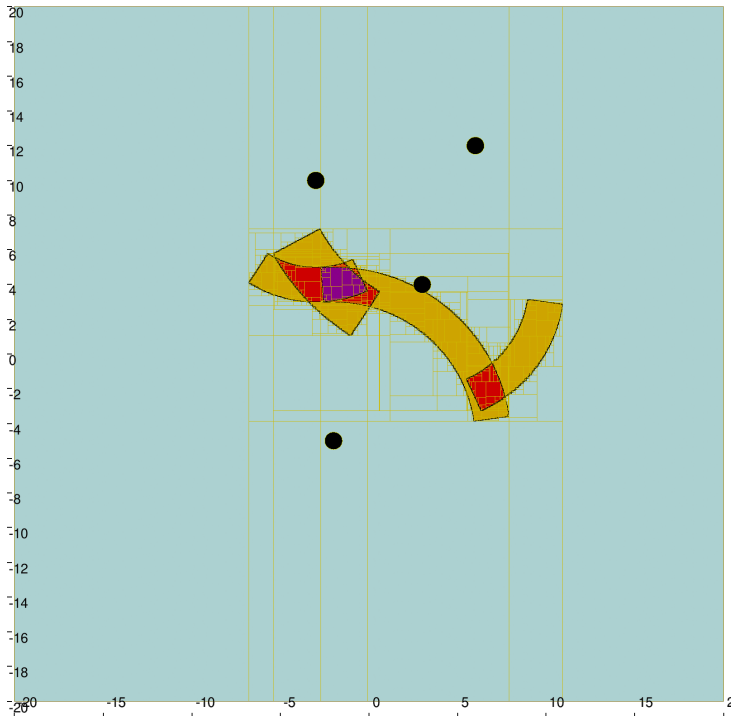

FIGURE 4 – Forward contraction with four landmarks

In the figure, the black dots are the landmarks. With each landmark we wrote the distance constraint. In yellow are the areas that satisfy at least two constraints, and in purple, all the locations that satisfy all four of the distance constraints (as we have four landmarks). This means that in the purple area, we are almost sure that the robot is here. In yellow it might be there, but it's less likely because not all the constraints agree on those locations. Red is intermediate.

Another efficient constraint in the problem of robot localisation, that I mostly used during my internship, is the TDoA constraint (Time Differential of Arrival). Thanks to [6], the TDoA constraint has proved to be very efficient in a computational point of view thanks to the Karush-Kuhn-Tucker equations. Because this constraint works as the time differential of arrival of two signals, the robot needs at least two landmarks to be able to use the TDoA constraint, and at least three landmarks to estimate its location with enough precision. The equation for the TDOA constraint is :

$$||x\breve{}a|| - ||x\breve{}b|| = y \tag{2}$$

where $x \in \mathbf{R^2}$ and $y \in \mathbf{R}$ are the variables. In our case, they represent the robot's position.

The parameters $a \in \mathbf{R^2}, b \in \mathbf{R^2}$ are assumed to be known as they represent the 2D positions of the two landmarks. In the annexes, I put the codes for building the TDoA constraint, both using a classic approach, and the KKT conditions. Here is how the TDoA constraint looks like. The dots represent the landmarks, and the green area is all the possible locations for the robot according to this constraint :

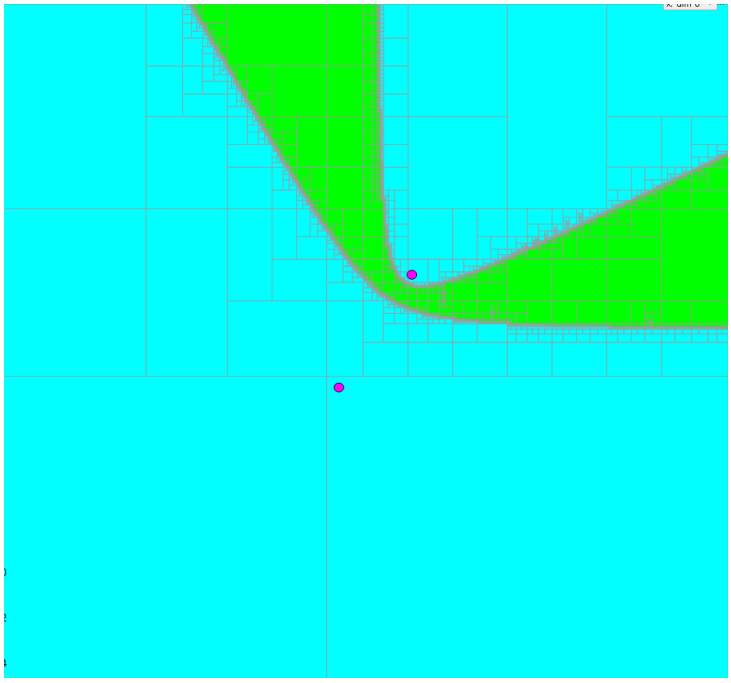

FIGURE 5 – TDoA constraint using the KKT conditions

The area is quite big and seems to diverge on the sides. That's why we have to combine various TDoA constraints together.

4.2 Model of fuzzy intervals

In order to model the measurements made by the robot and turn them into useful fuzzy intervals for our problem, we use a type of fuzzy numbers that are called crisp intervals. (see [4])

Let $\mathcal{A} = \{A_1, ..., A_n\}$ be a set of intervals. Then the degree of membership of the set captures the number of intervals that are overlapping for a given value $x \in \mathbf{R}$. In other words, for any value x, we can simply count the number of intervals in which x resides, computing the membership function as :

$$\mu_{\mathcal{A}}(x) = \frac{\sum_{i=1}^{n} \mu_{A_i}(x)}{n} \tag{3}$$

with

$$\mu_{A_i} = \begin{cases} 1 & l_{A_i} \leq x \leq r_{A_i}, \\ 0 & \text{else.} \end{cases}$$

The second simple formulation is convenient if one wishes to compute all the locations in the membership function where transitions from one alpha level α_i to either α_{i-1} or α_{i+1} occur. The membership function can be calculated as :

$$\mu_{\mathcal{A}}(x) = \frac{1}{n}\left(\sum_{i=1}^{n}(l_{A_i} \leq x) - \sum_{i=1}^{N}(r_{A_i} \leq x)\right) \tag{4}$$

This gives the membership of $\mathcal{A}$ at x by counting the number of left endpoints in $\mathcal{A}$ less than x and subtracting the number of right endpoints in $\mathcal{A}$ less than x. Thus, let $X_{\mathcal{A}} = \{x_1, ..., x_{2n}\}$ be the set of sorted interval endpoints, where $x_1 \leq x_2 \leq ... \leq x_{2n}$. From this set, we can build a fuzzy set that will look like this :

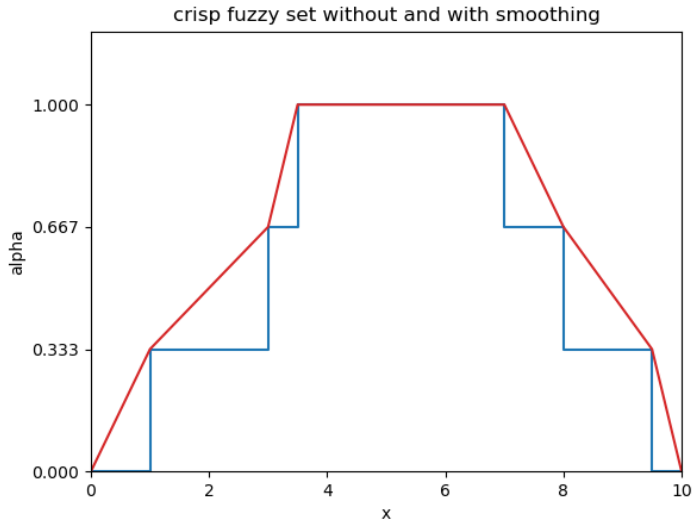

FIGURE 6 – transforming a crisp fuzzy set into a polygonal fuzzy set

4.3 Final results

To model the distance between a robot and a landmark I used these crisp fuzzy numbers. Let's assume that the robot holds two sensors (possibly more). Both sensors are from different models, with different precision and liability. After measurement, we obtain two different intervals that supposedly contain the 'true value' of the distance. From this data we build a crisp interval following the method mentioned previously, thus giving a polygonal fuzzy number :

In this configuration, the membership function equals to one when all the intervals overlap, which eventually occurs, because every interval of the set A at least contains the true value of the distance measure. In practice, it can happen all the constraints don't overlap due to the unliability of some sensor, or due to a to high number of constraints. In a computational point of view, we need to create a function that takes a set of intervals A as input and gives back another set B containing the scales of the membership function as intervals. Then another function will be required to generate the 'smoothed' polygonal fuzzy number. Then for any value of $\alpha \in [0, 1]$ we can compute the value an interval that will be used in the SIVIA Algorithm. For $\alpha = 1$ we have an interval with the smallest width possible, meaning that we have a high precision estimation of the value of the measurement. But we loose in liability. On the contrary, with $\alpha = 0$, We compute a very low precision interval of the measurement, but we increase the liability of the 'true value' being held inside the interval. In terms of displaying the results, we can put in the same figure the areas computed by the SIVIA Algorithm for several alpha cuts at the same time :

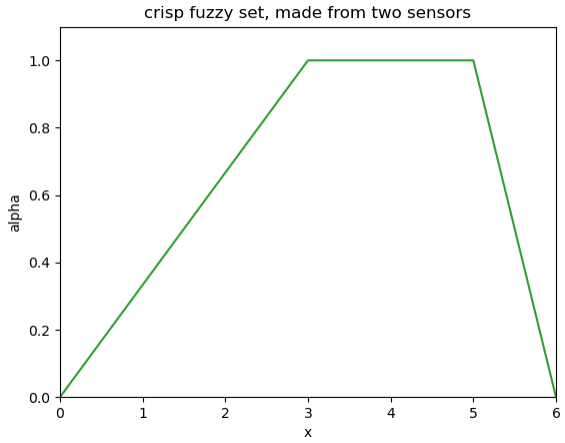

FIGURE 7 – crisp fuzzy set, made from two sensors

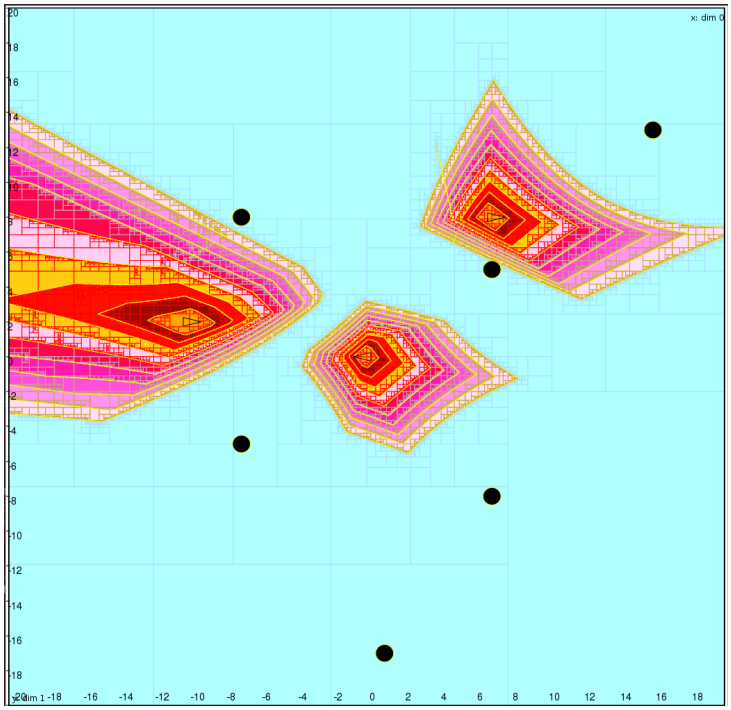

FIGURE 8 – 10 alpha cuts

In different shades of pink and red, we see the superposition of the results computed each time with different values of alpha. The higher the value of alpha, the smaller the region is. In this figure we have three robots.

5 Reflection on plagiarism

The previous section was originally written in my article. I used parts that I found most relevant for the report, and tried to re sentence a few lines, and to remodel the global layout so

the reader could understand better. What I did here shouldn't be a problem, since what I copied is my own article. But there actually was a time, when writing my article, where I wondered to what extent I could use my supervisor's articles inside mine. In this section we'll talk about my thoughts on plagiarism in different domains such as research, music or comedy, and try to look for the limits of plagiarism.

5.1 A quick definition of plagiarism

Whether we talk about research, music, comedy, we often hear polemics of artists or scientists that are said to have copied the work of an other. In research, recently, we heard about the french astrophysicist Etienne Klein who seemingly had plagiarized other scientists in his thesis [1]. He is said to have used their definitions, results and ideas without citing them, and without even changing the formulation. This is considered plagiarism in the scientific sphere because of the notion of intellectual property. Etienne Klein appropriated some property that was not his. According to [3], plagiarism is the "appropriation of another person's ideas, processes, results, or words without giving appropriate credit". Which means that in research, citing the work of another in one's own article is not considered plagiarism. And the scientific community is open enough to understand that in order to make progress in science, it is rudimentary to read and get inspiration/ideas from the work of others.

5.2 Plagiarism in music

Now plagiarism in other contexts is a bit more complex to bind. In music for instance, when an artist releases a song with the same melody or chord progression as an other already existing song, people (streamers, editors...) tend to make accusations of plagiarism. But defining intellectual property in the frame of music is a very tough thing to do. It is very likely that two different artists will come up with the same ideas in music. And generally, when a party sues the one that supposedly plagiarized, the accused can generally display a list of other musics that use the same melody or chord progression, so he can prove that it is not plagiarism. More generally, it is very common in music to take an existing song and make a cover or a remix. If the artist specifically announces the name of the author and the original song, it may not be considered plagiarism, but in fact that artist will still make money using intellectual property that isn't his. The musical industry is a bit unbalanced in this situation, because in order to avoid conflict, the artists create contracts known as royalties with one another. In this case, intellectual property is very hard to define. That is why there exist an anti-plagiarism cell which is made to answer this difficult question. (see [2] for further information about it). In other forms of art such as comedy, plagiarism is even more roughly seen. If an artist takes the jokes of another comedian and uses them while on scene, he will be 'canceled' by his fellow comedians. And it is very infrequent to have a comedian announce that he will make a joke that was invented by another before making said joke. In the same way as in music, taking a joke and citing it implies that the comedian will still make money thanks to a joke he did not invent. In this situation, the inherent rules of plagiarism are even more strict than in music or research.

5.3 To conclude

Coming back to my article, and thinking in the frame of research, even though I did use some definitions theorems and concepts that were written by my supervisors, the fact that I cited them properly makes it non plagiarism in my case. Those part actually represent a fair percentage of my article, but it was necessary in order to have perfectly formulated concepts, and for the rest of my article to be understandable. Besides, I wrote the article having in mind that I was co-authoring it with Mr Arana Jiménez and Mr Jaulin. Which is why in terms of legitimacy, my article shouldn't have any problems with plagiarism at all.

6 Critical Analysis and reflection

This internship came with its fair share of teachings and paths of improvements. I learned a lot about research, mathematics and interval analysis. My English and my Spanish improved greatly, and I also got better at using C++, CmakeLists and the CODAC framework. I also made mistakes that any engineer should learn to avoid. I will talk about one that consumed a lot of time and efforts. I wanted to develop a program that would build a list of contractors, and from that list, it would take as input a number of contractors n (inferior to the length of the list) and build a new list of all possible combinations of n contractors. This would help, after computation to use all the available landmarks to have the best estimation possible. So I created three C++ functions, it took time because of the complexity of the algorithm. Once I finished and went through the process of debugging, I realized, by reading the CODAC documentation that this function already existed. It was built in the framework, and not knowing that cost me a few days of work. Here is what my program looks like, compared to what it looked like after using the CODAC builtin function :

```
vector<Array<Sep>>* Combinaisons(vector<Sep*> seps, int alpha){
    int N = seps.size();

    if (N == alpha){
        vector<Array<Sep>>* test;
        test = new vector{Array<Sep> { vec: seps}};
        return test;
    }
    else if (alpha == 0){
        vector<Array<Sep>>* test;
        test = new vector{Array<Sep> {}};
        return test;
    }

    Sep* first_element;
    first_element = seps[0];

    vector<Sep*> sous_seps;
    sous_seps.insert( position: sous_seps.begin(), first: seps.begin() + 1, last: seps.end());
    vector<Array<Sep>>* comb_with_first = Combinaisons( seps: sous_seps, alpha: alpha - 1);
    for (int k = 0; k < comb_with_first->size(); k++){
        (*comb_with_first)[k].add( &: (*first_element));
    }

    vector<Array<Sep>>* comb_without_first = Combinaisons( seps: sous_seps, alpha);

    vector<Array<Sep>>* comb = comb_with_first;
    for (int k = 0; k < comb_without_first->size(); k++){
        comb->push_back((*comb_without_first)[k]);
    }
    //comb.insert(comb.end(), comb_without_first.begin(), comb_without_first.end());
    //comb.insert(comb.end(), comb_without_first);
    return comb;

}
```

FIGURE 9 – The function I wrote

```
SepQInterProjF sepFinal0( list: sepArray, q: 0);
```

FIGURE 10 – The builtin function

This experience taught me to pay more attention to the documentation, that programmers are lucky to have access to those documents. I should always check the documentation before

going head first into programming. It can sometimes save a few hours of work.

7 Consequences of this internship on my career plan

This internship was a great opportunity for me to find out whether I may work in research later on or not. When I was younger, I often thought about becoming a physicist, even though it was not on a robotics field, I wanted to become an astrophysicist. For many years I was really motivated, I did some observation internships and read a lot about the matter. But after my years in CPGE, I lost taste in studies and decided during my first year at ENSTA Bretagne that I would just finish my 3 years of engineering school, then have my diploma and work in a company. But then I chose to specialize in submarine robotics, and the subject started to raise my interest. All the projects and conception that we do in 2nd and 3rd year helped to rekindle my motivation for research. And I think this internship at INDESS helped me realize that I have the ability to work in research. But the subject of my potential thesis has to interest me well enough. For instance I know that I don't want to work on Interval Analysis anymore. Currently, I would like to end my studies and start working, partly to obtain my financial independence, but having two big brothers that work as engineers, I have a rough taste of what they do and I'm not sure whether I would like it or not (even though I am conscious that they don't work on robotics). I think working in Research and Development for a company would benefit me more. In other words, I still don't know for now if I want to pursue my studies with a thesis or not. And I am waiting for my end of studies project (hopefully in a company this time) to decide what I want to do in the future.

8 Conclusion

This internship was particularily interesting thanks to the subject, interval analysis, fuzzy intervals and programming, but also of cours thanks to the questions and thinkings that it raised. I could make a step forward in my decision of whether I want to make a thesis or not, and I got to reflect on the notion of plagiarism. Last but not least, I had the occasion to write a science paper which I consider a big opportunity.

9 Thanks

I would like to warmly thank my supervisors, Mr Arana-Jiménez and Mr Jaulin, for their patience and their guidance throughout the internship. I learnt a whole lot of things thanks to them. I especially thank Mr Arana-Jiménez for welcoming me in the office of INDESS, thus helping me feel at the right place, and also for granting me the opportunity to write an article.

A Code Snippets

Références

[1] Science et Avenir. Etienne klein : « je ne démissionnerai pas de la présidence de l'ihest ». In *https ://www.sciencesetavenir.fr/fondamental/etienne-klein-accuse-de-plagiat-je-ne-demissionnerai-pas-de-la-presidence-de-l-ihest*$_1$08540, 2016.

[2] Etienne Guerault. Plagiat ou influence ? In *https ://youtu.be/F1Voe*$_A$*RoZg?si* = $6Y8yd6s5QeiwoVqJ$, 2023.

[3] Farrokh Habibzadeh and Karen Shashok. Plagiarism in scientific writing : words or ideas ? In *https ://www.ncbi.nlm.nih.gov/pmc/articles/PMC3160704/*, 2011.

```
void sivia_TDoA_Sy(IntervalVector X, vector<vector<double>> M, ibex::SepUnion Sy, double epsi){
    vector<CtcTdoaFwdKKT> Cfwd;
    vector<CtcTdoaBwdClassic> Cback;
    for (int i = 0; i < Sy.nb_var; i ++){
        Cfwd.push_back(CtcTdoaFwdKKT( &: M[i],  &: M[i+1]));
        Cback.push_back(CtcTdoaBwdClassic( &: M[i],  &: M[i+1]));
    }
    SepAct S( &: Cfwd,  &: Cback,  &: Sy);
    SIVIA(X,  &: S,  precision: 0.01);
}
```

FIGURE 11 – Function for a TDoA constraint

```
Array<Sep> sepArray{ &: *sepstdoa[0],  &: *sepstdoa[1],  &: *sepstdoa[2],  &: *sepstdoa[3]};
SepQInterProjF sepFinal0( list: sepArray,  q: 0);
SepQInterProjF sepFinal1( list: sepArray,  q: 1);
SepQInterProjF sepFinal2( list: sepArray,  q: 2);
//SepQInterProjF sepFinal3(sepArray, 3);
vector<SepQInterProjF> SEPS{sepFinal2, sepFinal1, sepFinal0};

SetColorMap col10{{ x: SetValue::IN,  y: "#CCAA33[#FF00FF22]"}, { x: SetValue::OUT,  y: "#AADDFF[#AAFFFFEE]"}, { x: SetValue::UNKNOWN,  y: "yellow[white]"}};
SetColorMap col7{{ x: SetValue::IN,  y: "#CCAAAA[#FF0099AA]"}, { x: SetValue::OUT,  y: "transparent"}, { x: SetValue::UNKNOWN,  y: "yellow[white]"}};
SetColorMap col3{{ x: SetValue::IN,  y: "#FF0000FF[#FF0000EE]"}, { x: SetValue::OUT,  y: "transparent"}, { x: SetValue::UNKNOWN,  y: "yellow[white]"}};
vector<SetColorMap> col{col10, col7, col3};

vector<string> ImageNames{"../Documents/ENSTA/stage2A/tdoa1.png", "../Documents/ENSTA/stage2A/tdoa2.png", "../Documents/ENSTA/stage2A/tdoa3.png","../Documents/ENSTA/stage2A/tdoa4.
// generating the images, using the separators in SIVIA for different alpha cuts

vibes::beginDrawing();
IntervalVector P( n: 2,  x: Interval( a: -20,  b: 20));
VIBesFigMap Map( fig_name: "tdoa");
for (int alpha = 0; alpha < sepstdoa.size()-1; alpha++){

    SIVIA( x: P,
           &: SEPS[alpha],
           precision: 0.1,
           regular_paving: true,
           display_result: true,
           fig_name: "tdoa",
           return_result: false,
           color_map: col[alpha]);

    for (int k = 0; k < M.size(); k++){
        vibes::drawCircle( cx: M[k][0].lb(),  cy: M[k][1].ub(),  r: 0.5,  format: "yellow[black]");
    }

    Map.set_properties( x: 100,  y: 100,  width: 800,  height: 800);
    vibes::saveImage( fileName: ImageNames[alpha],  figureName: "tdoa");

}
```

FIGURE 12 – main for using a number n of landmarks

```
14    double phi1(double x1, vector<double> a, vector<double> b){
15        double a1 = a[0];
16        double a2 = a[1];
17        double b1 = b[0];
18        double b2 = b[1];
19        //Interval X1(x1, x1);
20        double value = (a2*abs( x: x1-b1)-b2*abs( x: x1-a1))/(abs( x: x1-b1)-abs( x: x1-a1));
21        return value;
22    }
23
24
25    double phi2(double x2, vector<double> a, vector<double> b){
26        return phi1( x1: x2,  a: {a[1], a[0]},  b: {b[1], b[0]});
27    }
```

FIGURE 13 – Functions used in the building of the constraint using KKT conditions

```
52  CtcTdoaFwdKKT::CtcTdoaFwdKKT() : Ctc( n: 3) {}
53
54  CtcTdoaFwdKKT::CtcTdoaFwdKKT(vector<double>& a, vector<double>& b) : Ctc( n: 3){
55      _a = a;
56      _b = b;
57  }
58
59  void CtcTdoaFwdKKT::contract(IntervalVector& X){
60      if (X.is_empty()){
61          X = IntervalVector( n: 3,  x: Interval::EMPTY_SET);
62      }
63      else{
64          Interval X1 = X[0];
65          Interval X2 = X[1];
66          vector<double> F{f( x1: X1.lb(),  x2: X2.lb(),  a: _a,  b: _b), f( x1: X1.lb(),  x2: X2.ub(),  a: _a,  b: _b), f( x1: X1.ub(),  x2: X2.lb(),  a: _a,  b: _b), f( x1: X1.ub(),  x2: X2.ub(),  a: _a,
67          auto minimum : iterator<...>  = std::min_element( first: F.begin(),  last: F.end());
68          auto maximum : iterator<...>  = std::max_element( first: F.begin(),  last: F.end());
69          Interval F2( a: *minimum,  b: *maximum);
70
71          vector<double> XX1{X1.lb(), X1.ub()};
72          for (double x1 : XX1){
73              if (not X2.is_disjoint( x: phi1(x1,  a: _a,  b: _b))){
74                  F2 = F2|f(x1,  x2: phi1(x1,  a: _a,  b: _b),  a: _a,  b: _b);
75              }
76          }
77          vector<double> XX2{X2.lb(), X2.ub()};
78          for (double x2 : XX2){
79              if (not X1.is_disjoint( x: phi1( x1: x2,  a: _a,  b: _b))){
80                  F2 = F2|f( x1: x2,  x2: phi2(x2,  a: _a,  b: _b),  a: _a,  b: _b);
81              }
82          }
83          X[2] = X[2]&F2;
84      }
85  }
```

FIGURE 14 – Class for KKT constraint using TDoA

textwidthtextwidth

FIGURE 15 – Class for Classic constraint using TDoA

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