ENSTA Bretagne

Lab-STICC

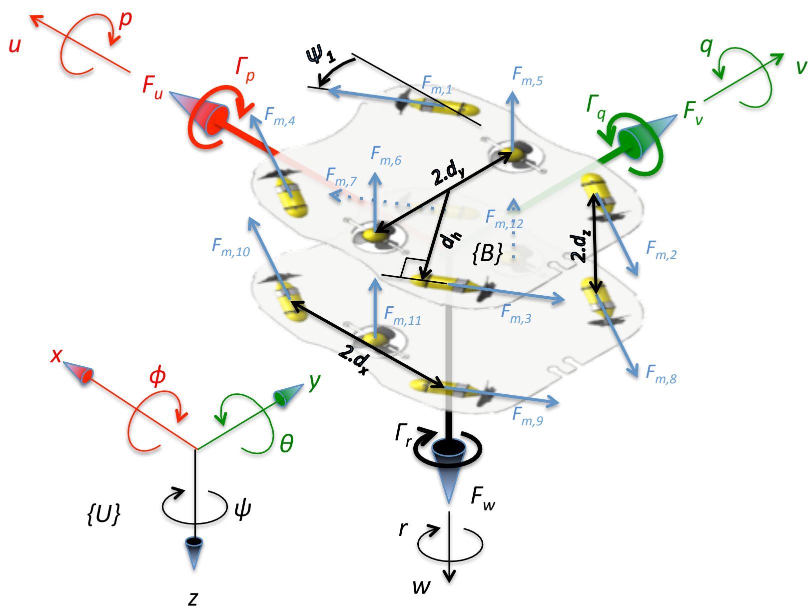
Redundancy Management

Robex-Day 5/12/2023

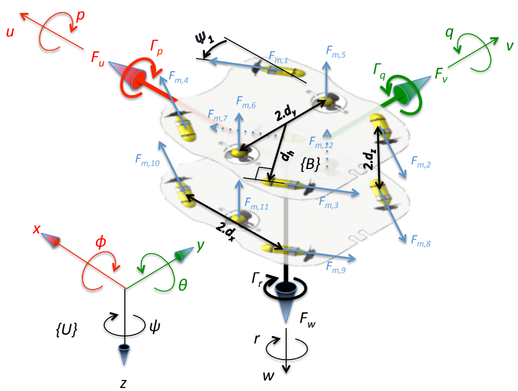
Redundancy

- Redundant actuation system
  - System that has more actuators than degrees of freedom
  - A system can have a redundant actuation system, while remaining under-actuated
    - Only subsets of Dofs can be redundant
  - Opportunities in the control allocation problem

$$\mathbf{F}_\mathrm{B} = \underbrace{\mathbf{A}}_{\text{‘Concentrator’}} \cdot \mathbf{F}_\mathrm{m}$$



Redundancy



- The Control Allocation problem :
  - Given the actuation system :

$$\mathbf{F}_\mathrm{B} = \underbrace{\mathbf{A}}_{\text{'Concentrator'}} \cdot \mathbf{F}_\mathrm{m}$$

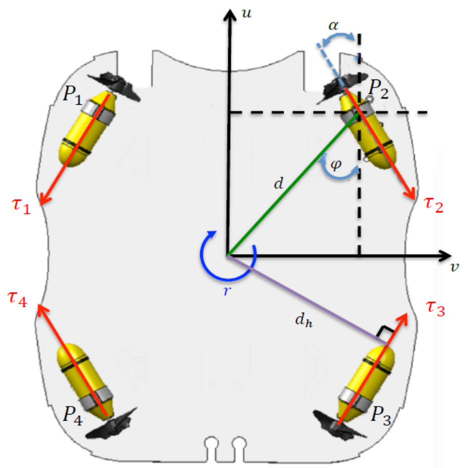
  - Compute the desired actuators' force, $\mathbf{F}_\mathrm{m}^\mathrm{d}$, in order to produce a prescribed resulting action $\mathbf{F}_\mathrm{B}^\mathrm{d}$ on the system.

$$\mathbf{F}_\mathrm{m}^\mathrm{d} = \underbrace{\mathbf{D}}_{\text{'Dispatcher'}} \cdot \mathbf{F}_\mathrm{B}^\mathrm{d}$$

    - If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} > n$, under-determined
    - If $\det(\mathbf{A}) \neq 0$, $\mathbf{D}$ is unique and $\mathbf{D} = \mathbf{A}^{-1}$
    - If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} < n$, $\mathbf{D}$ is not unique

Redundancy

- Example : Jack
  - 6 actuators for 6 Dof
  - redundant in the H plane
  - Globally underactuated

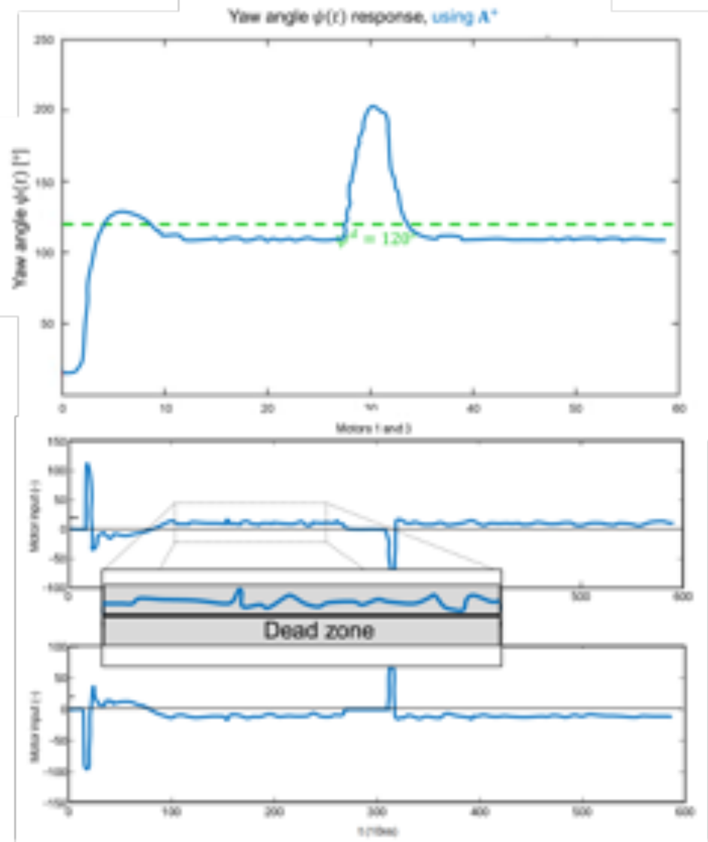


$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}} = \underbrace{\begin{bmatrix} -\cos\varphi & -\cos\varphi & \cos\varphi & \cos\varphi \\ -\sin\varphi & \sin\varphi & \sin\varphi & -\sin\varphi \\ d & -d & d & -d \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{\mathbf{F_m}}$$

$$\mathbf{F_B} = \mathbf{A} \cdot \mathbf{F_m}$$

Redundancy

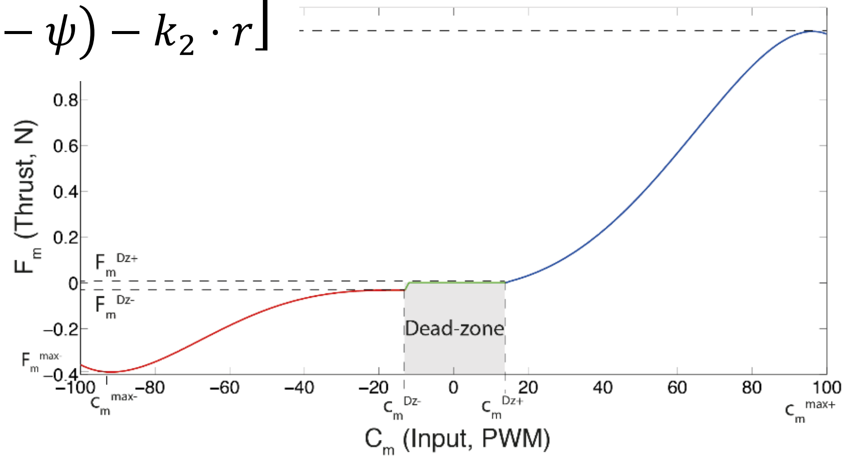
- Example : Jack, Dead-zone effects compensation
  - Use of the Moore-Penrose pseudo inverse : $\mathbf{A}^{+} = \mathbf{A}^{\mathrm{T}} \cdot \left(\mathbf{A} \cdot \mathbf{A}^{\mathrm{T}}\right)^{-1}$



$$\mathbf{F}_{\mathrm{m}} = \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

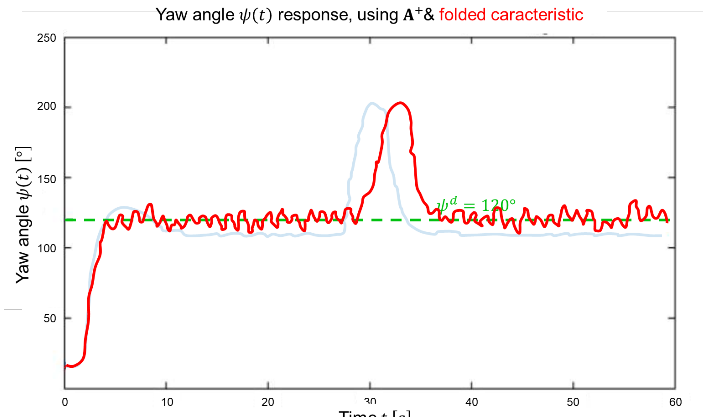
  - Test on Jack, yaw regulation

$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$



Redundancy

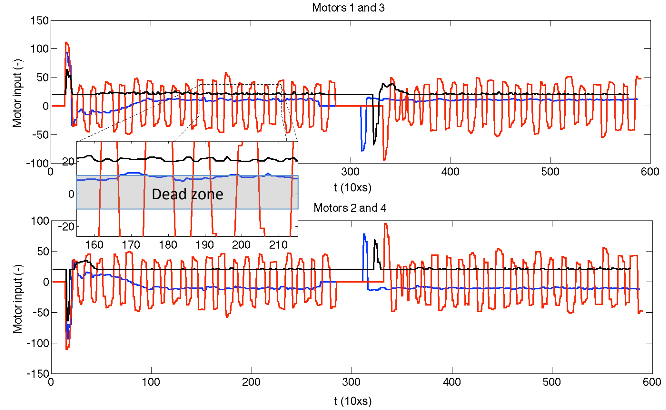
- Example : Jack, Dead-zone effects compensation
  - Use of the Moore-Penrose pseudo inverse : $\mathbf{A}^{+} = \mathbf{A}^{\mathrm{T}} \cdot \left(\mathbf{A} \cdot \mathbf{A}^{\mathrm{T}}\right)^{-1}$

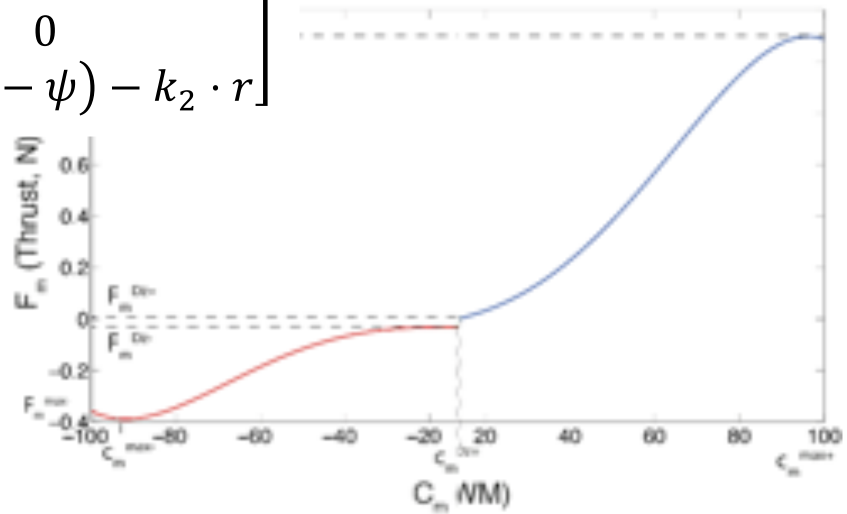


$$\mathbf{F}_{\mathrm{m}} = \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

  - Test on Jack, yaw regulation, folding DZ

$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$





Redundancy

- Example : Jack, Dead-zone effects compensation
  - Use null-space projetor

If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} > n$, $\rightarrow \ker \mathbf{A} \neq \{\emptyset\}$, and $\forall\, \mathbf{M}_{\mathrm{m}} \in \ker \mathbf{A}$, $\mathbf{A} \cdot \mathbf{M}_{\mathrm{m}} = \mathbf{0}$

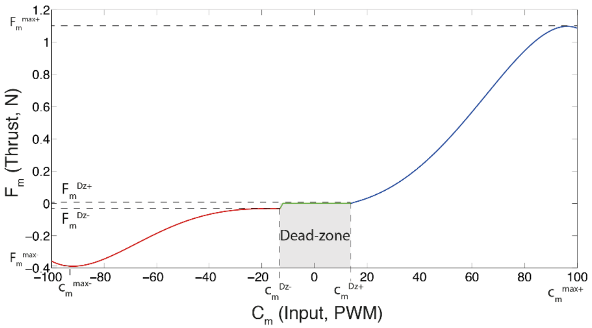
Hence, $\forall\, r_{\mathrm{m}} \in \mathbb{R}$, $\mathbf{F}_{\mathrm{m}} = \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \mathbf{M}_{\mathrm{m}} \cdot r_{\mathrm{m}} \rightarrow \mathbf{F}_{\mathrm{B}} = \mathbf{A} \cdot \mathbf{F}_{\mathrm{m}} = \mathbf{A} \cdot \left(\mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \mathbf{M}_{\mathrm{m}} \cdot r_{\mathrm{m}}\right)$

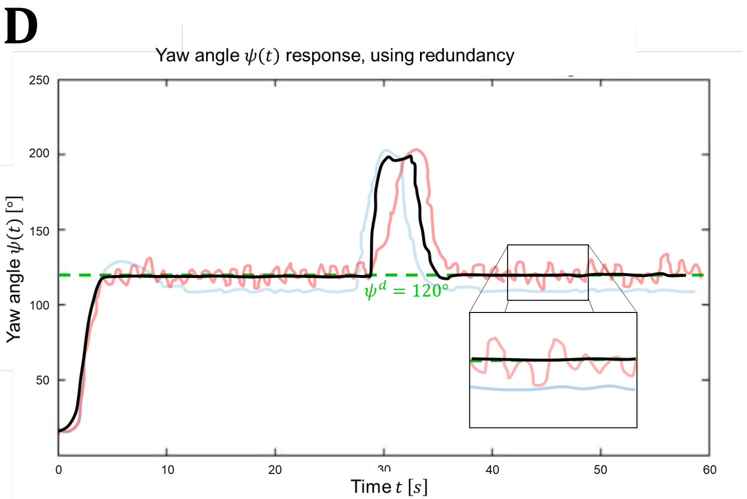
$$= \mathbf{A} \cdot \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \mathbf{A} \cdot \mathbf{M}_{\mathrm{m}} \cdot r_{\mathrm{m}} = \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

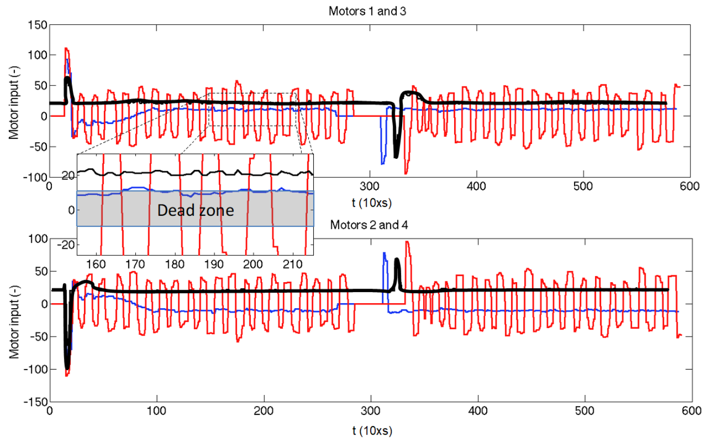
$$= \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$ (for some nice properties of $\mathbf{A}$)

$$\mathbf{F}_{\mathrm{m}} = \underbrace{\begin{bmatrix} \mathbf{A}^{+} & \mathbf{M}_{\mathrm{m}} \end{bmatrix}} \cdot \begin{bmatrix} \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} \\ r_{\mathrm{m}} \end{bmatrix}$$

$$\mathbf{M}_{\mathrm{m}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, r_m = 20\%$$



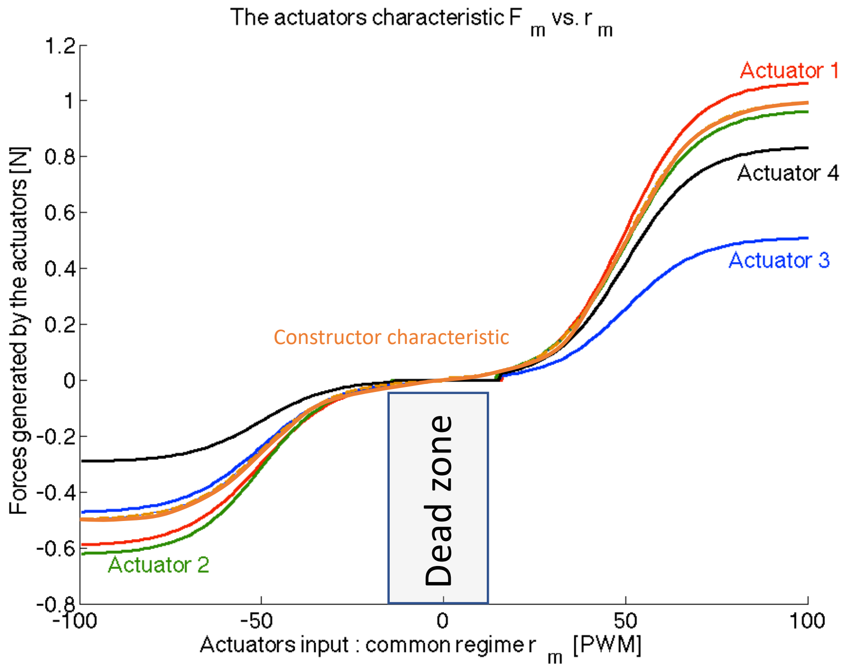




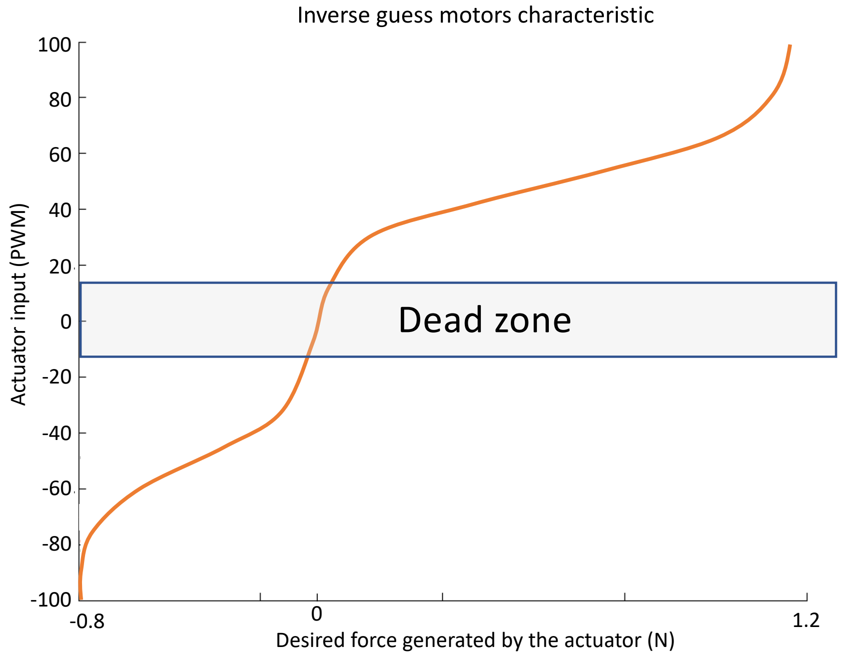
Redundancy

- Example : Robustness against motors' characteristic uncertainty and disparity

$$\mathbf{F_m} = \Omega(\mathbf{c_m})$$



$$\mathbf{c_m} = \hat{\Omega}^{-1}(\mathbf{F_m})$$



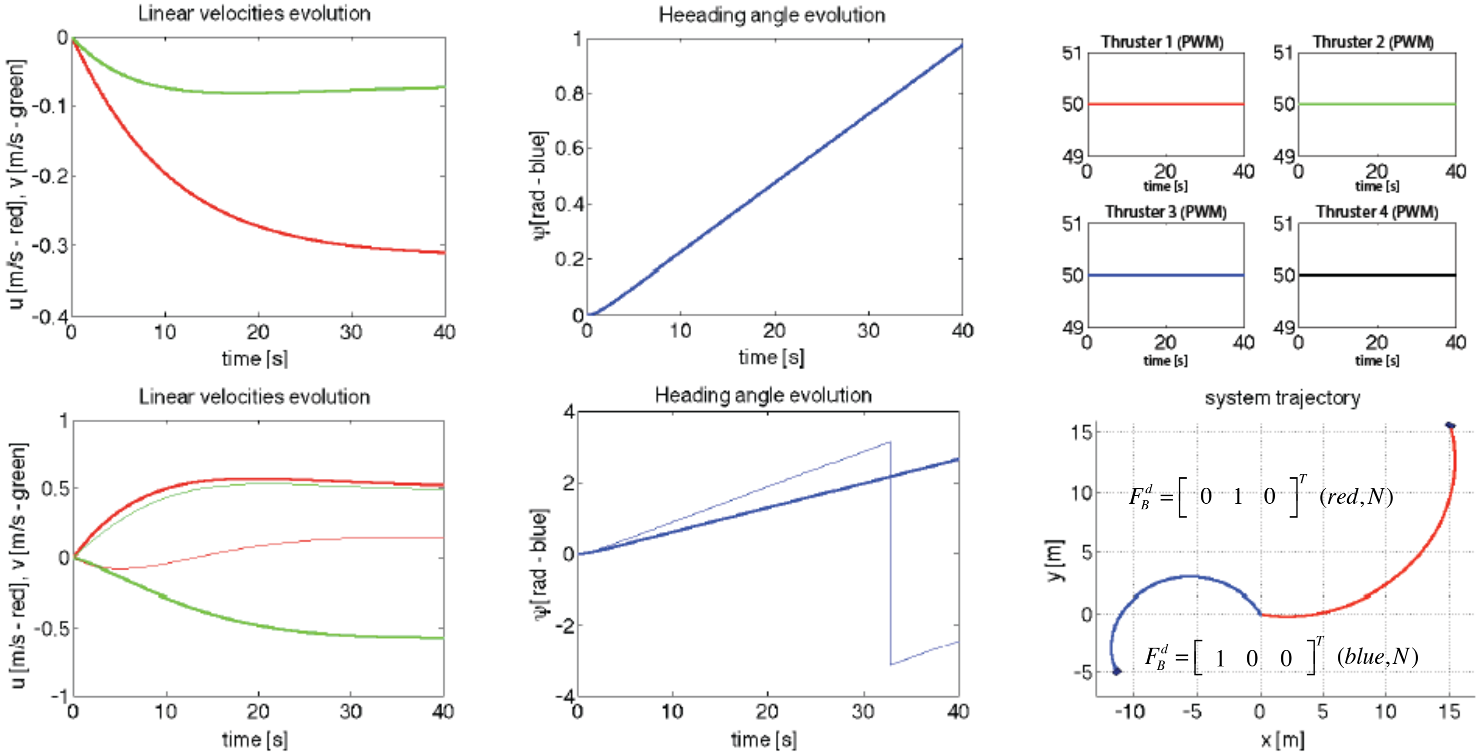
$$\mathbf{F}_B = \mathbf{A} \cdot \Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A^+} \cdot \mathbf{F_B^d} + \mathbf{M_m} \cdot r_m\right)\right) \equiv \mathbf{A} \cdot \Omega \cdot \hat{\Omega}^{-1} \cdot \left(\mathbf{A^+} \cdot \mathbf{F_B^d} + \mathbf{M_m} \cdot r_m\right) \neq \mathbf{F_B^d}$$

➔ DOF Coupling effect

Redundancy

- Example : Robustness against motors' characteristic uncertainty and disparity

Open loop



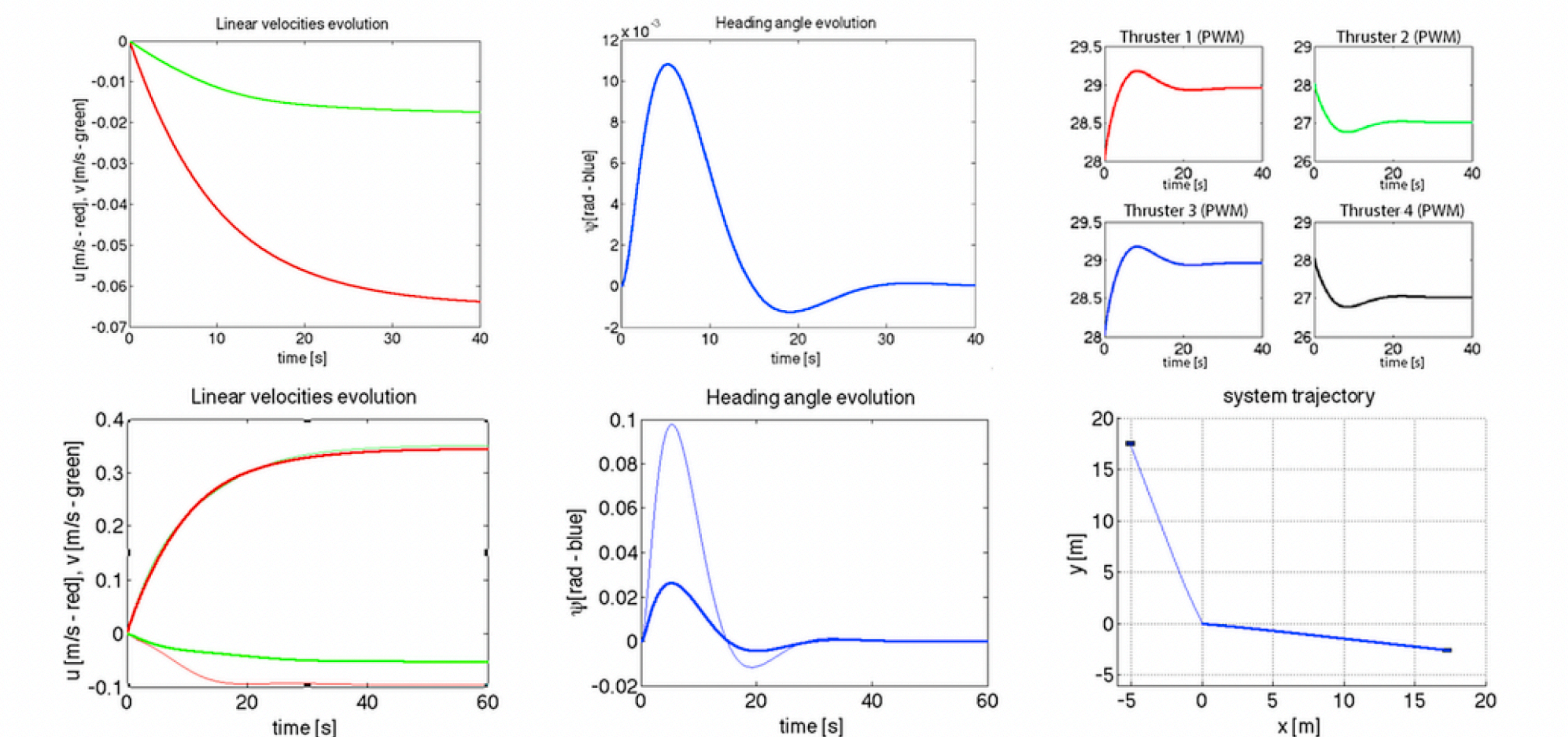
$$\mathbf{F}_B = \mathbf{A} \cdot \Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+} \cdot \mathbf{F}_{\mathbf{B}}^{\mathbf{d}} + \mathbf{M}_{\mathbf{m}} \cdot r_m\right)\right) \equiv \mathbf{A} \cdot \Omega \cdot \hat{\Omega}^{-1} \cdot \left(\mathbf{A}^{+} \cdot \mathbf{F}_{\mathbf{B}}^{\mathbf{d}} + \mathbf{M}_{\mathbf{m}} \cdot r_m\right) \neq \mathbf{F}_{\mathbf{B}}^{\mathbf{d}}$$

➔ DOF Coupling effect

Redundancy

- Example : Robustness against motors' characteristic uncertainty and disparity

With yaw regulation



$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+}\cdot\mathbf{F}_{\mathbf{B}}^{\mathbf{d}}+\mathbf{M}_{\mathbf{m}}\cdot r_m\right)\right)\equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^{+}\cdot\mathbf{F}_{\mathbf{B}}^{\mathbf{d}}+\mathbf{M}_{\mathbf{m}}\cdot r_m\right)\neq \mathbf{F}_{\mathbf{B}}^{\mathbf{d}}$$

➔DOF Coupling effect

Redundancy

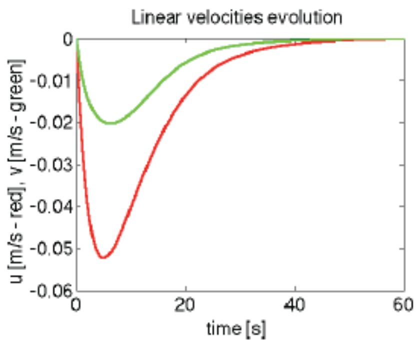
- Example : Robustness against motors' characteristic uncertainty and disparity

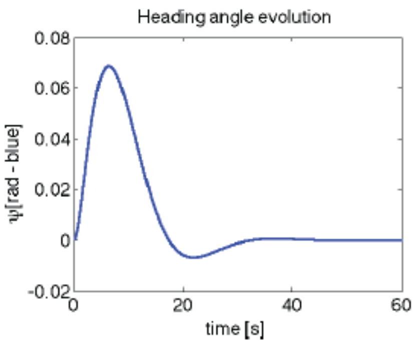
$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+}\cdot\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right)\right) \equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^{+}\cdot\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right) \neq \mathbf{F}_\mathbf{B}^\mathbf{d}$$

➔ DOF Coupling effect

- Identification of corrective coefficients
  - Consider the following regulation law :

$$\mathbf{F}_\mathbf{B}^d = \begin{bmatrix} -u - \int_0^t u\cdot \mathrm{d}t \\ -v - \int_0^t v\cdot \mathrm{d}t \\ -\psi - r - 0.1\cdot\int_0^t \psi\cdot \mathrm{d}t \end{bmatrix}, \begin{cases} \mathbf{c_m} = \hat{\Omega}^{-1}\cdot\mathbf{A}^{+}\cdot\left(\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right) \\ r_m = \Omega\cdot c_0 \end{cases}$$

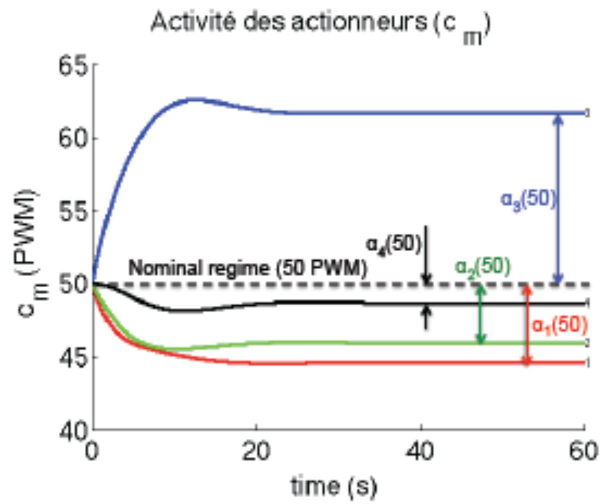




$$\mathbf{F}_\mathbf{B} = \mathbf{A}\cdot\Omega\cdot\mathbf{c}_\mathbf{m}^\infty(c_0) = \mathbf{0}$$

$$\Rightarrow \mathbf{c}_\mathbf{m}^\infty(c_0) \in \ker(\mathbf{A}\cdot\Omega)$$

$$\Rightarrow \alpha_i(c_0) = \frac{c_{\mathbf{m},i}^\infty}{c_0}$$



Redundancy

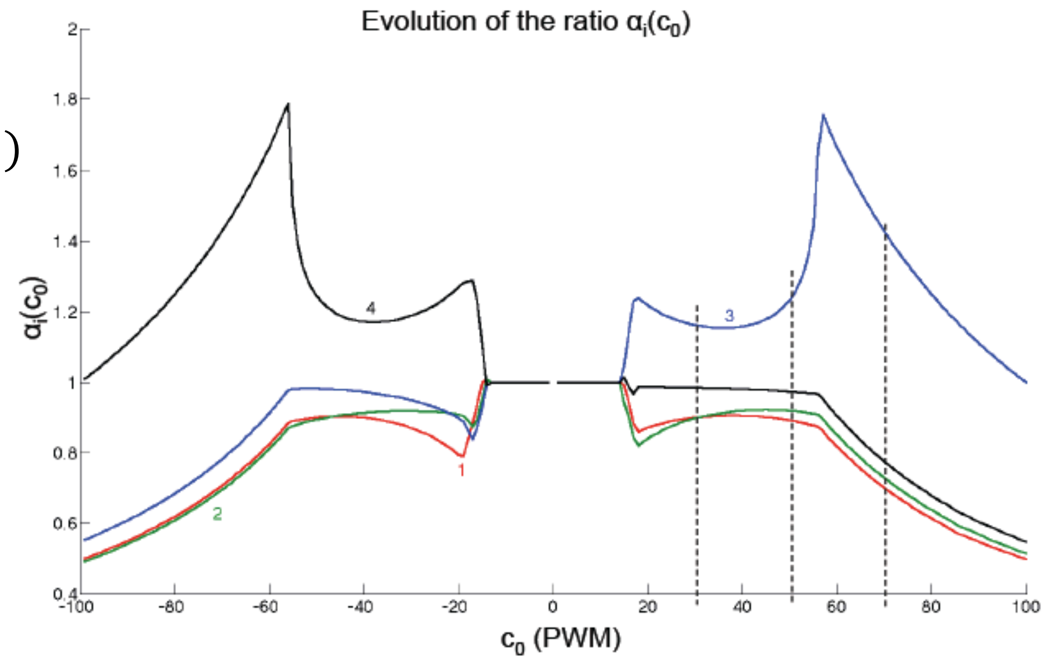
- Example : Robustness against motors' characteristic uncertainty and disparity

$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^+\cdot\mathbf{F}_\mathbf{B}^\mathbf{d}+\mathbf{M}_\mathbf{m}\cdot r_m\right)\right)\equiv\mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^+\cdot\mathbf{F}_\mathbf{B}^\mathbf{d}+\mathbf{M}_\mathbf{m}\cdot r_m\right)\neq\mathbf{F}_\mathbf{B}^\mathbf{d}$$

➔DOF Coupling effect

- Identification of corrective coefficients
  - Consider the following regulation law
  - Iterate for $c_\mathrm{m}^\mathrm{min} < c_\mathrm{m} < c_\mathrm{m}^\mathrm{max}$ and build $\mathbf{Q}(\mathbf{c}_\mathrm{m})$

$$\mathbf{F}_\mathbf{B} = \mathbf{A}\cdot\Omega\cdot\mathbf{c}_\mathbf{m}^\infty(c_0)=\mathbf{0}$$
$$\Rightarrow \mathbf{c}_\mathbf{m}^\infty(c_0)\in\ker(\mathbf{A}\cdot\Omega)$$
$$\Rightarrow \alpha_i\left(c_0\right)=\frac{c_{\mathbf{m},i}^\infty}{c_0}$$
$$\Rightarrow \mathbf{Q}(\mathbf{c}_\mathbf{m})=\mathrm{diag}(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$$



Redundancy

- Example : Robustness against motors' characteristic uncertainty and disparity

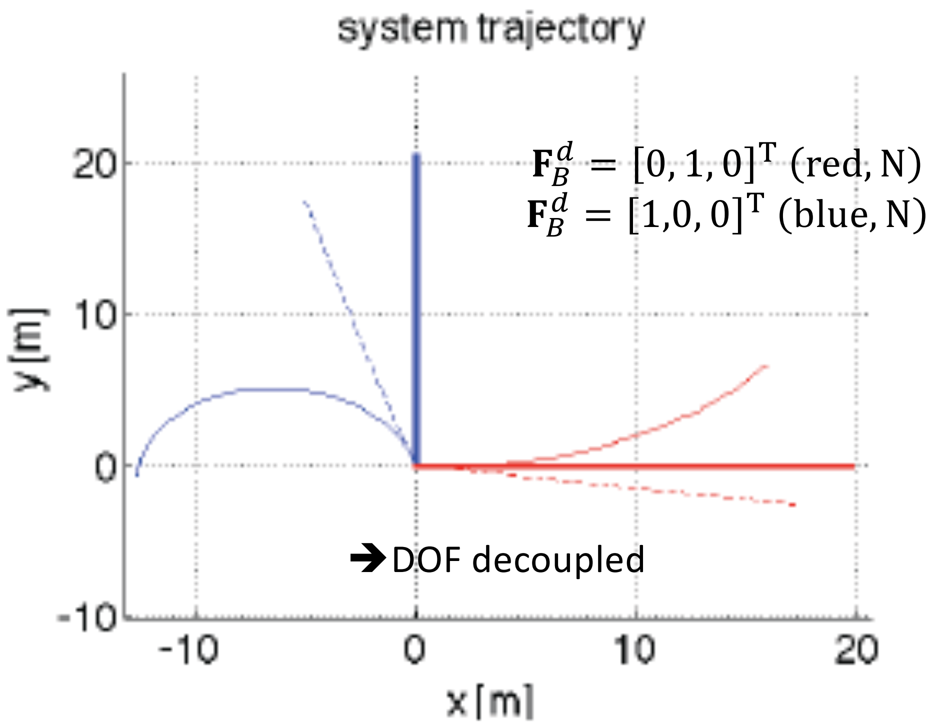
$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+}\cdot\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right)\right) \equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^{+}\cdot\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right) \neq \mathbf{F}_\mathbf{B}^\mathbf{d}$$

➔DOF Coupling effect

- Identification of corrective coefficients
  - Consider the following regulation law
  - Iterate for $c_\mathrm{m}^\mathrm{min} < c_\mathrm{m} < c_\mathrm{m}^\mathrm{max}$ and build $\mathbf{Q}(\mathbf{c}_\mathrm{m})$
  - Implement the follwing open loop control

$$\mathbf{c}_\mathbf{m} = \mathbf{Q}\left(\cdot\hat{\mathbf{\Omega}}^{-1}\cdot\mathbf{A}^{+}\cdot\left(\mathbf{F}_\mathbf{B}^\mathbf{d} + \mathbf{M}_\mathbf{m}\cdot r_m\right)\right)$$

$$\mathbf{F}_\mathbf{B} \equiv \mathbf{A}\cdot\mathbf{\Omega}\cdot\mathbf{Q}\cdot\hat{\mathbf{\Omega}}^{-1}\cdot\begin{bmatrix} \mathbf{A}^{+} & M_m \end{bmatrix}\cdot\begin{bmatrix} \mathbf{F}_\mathbf{B}^\mathbf{d} \\ r_m \end{bmatrix} = k_Q\cdot\mathbf{F}_\mathbf{B}^\mathbf{d}$$

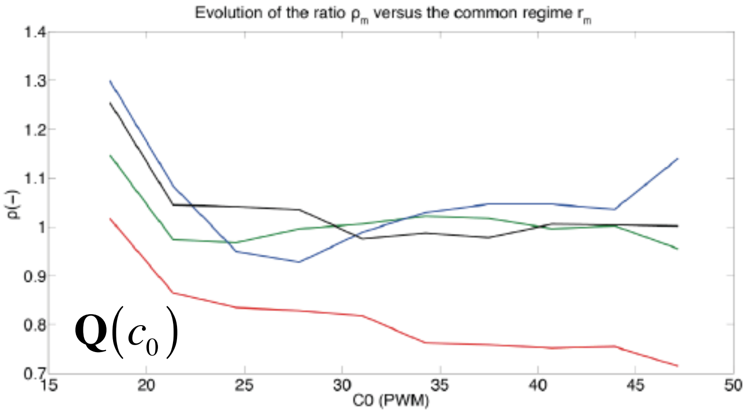


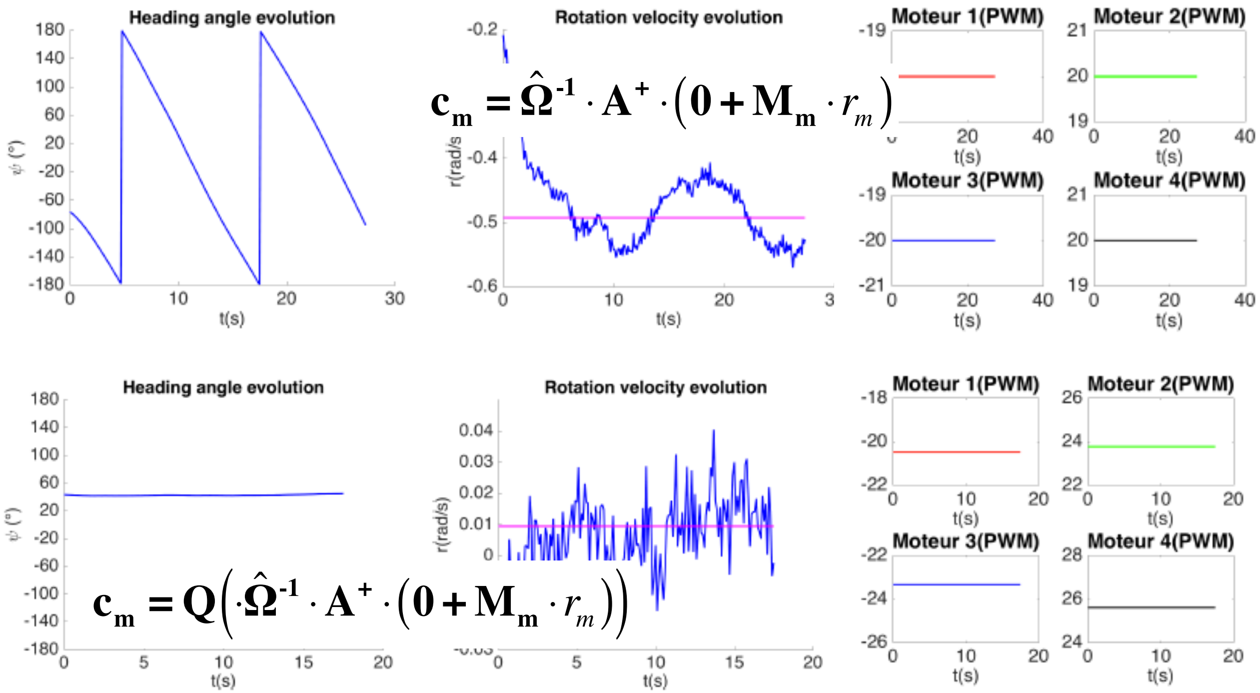
Redundancy

- Example : Robustness against motors' characteristic uncertainty and disparity

$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+}\cdot\mathbf{F}_{\mathbf{B}}^{\mathbf{d}} + \mathbf{M}_{\mathbf{m}}\cdot r_m\right)\right) \equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^{+}\cdot\mathbf{F}_{\mathbf{B}}^{\mathbf{d}} + \mathbf{M}_{\mathbf{m}}\cdot r_m\right) \neq \mathbf{F}_{\mathbf{B}}^{\mathbf{d}}$$ ➔ DOF Coupling effect

  - Identification of corrective coefficients
  - Experimentations





$$\mathbf{c}_{\mathbf{m}} = \hat{\boldsymbol{\Omega}}^{\mathbf{-1}}\cdot\mathbf{A}^{+}\cdot\left(\mathbf{0} + \mathbf{M}_{\mathbf{m}}\cdot r_m\right)$$

$$\mathbf{c}_{\mathbf{m}} = \mathbf{Q}\left(\cdot\hat{\boldsymbol{\Omega}}^{\mathbf{-1}}\cdot\mathbf{A}^{+}\cdot\left(\mathbf{0} + \mathbf{M}_{\mathbf{m}}\cdot r_m\right)\right)$$

Redundancy

- Dead-zone effects compensation
- Robustness against motors' characteristic uncertainty and disparity
- Robustness against actuators failure : $\mathbf{r}_{\mathrm{m}}$? / $F_{l=1,\dots,k} = 0$ & $\mathbf{F}_{\mathrm{B}} = [F_1, \dots, F_m]^{\mathrm{T}} = \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$

$$\underset{(6\times1)}{\mathbf{F}_{\mathrm{B}}} = \underset{(6\times12)}{\mathbf{A}} \cdot \underset{(12\times1)}{\mathbf{F}_{\mathrm{m}}}$$

$$\underset{(12\times1)}{\mathbf{F}_{\mathrm{m}}} = \underbrace{[\mathbf{A}^{+} \quad \mathbf{M}_{\mathrm{m}}]}_{\substack{\mathbf{D} \\ (12\times12)}} \cdot \underset{(12\times1)}{\begin{bmatrix} \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} \\ \mathbf{r}_{\mathrm{m}} \end{bmatrix}} \; (6\times1)$$

Redundancy

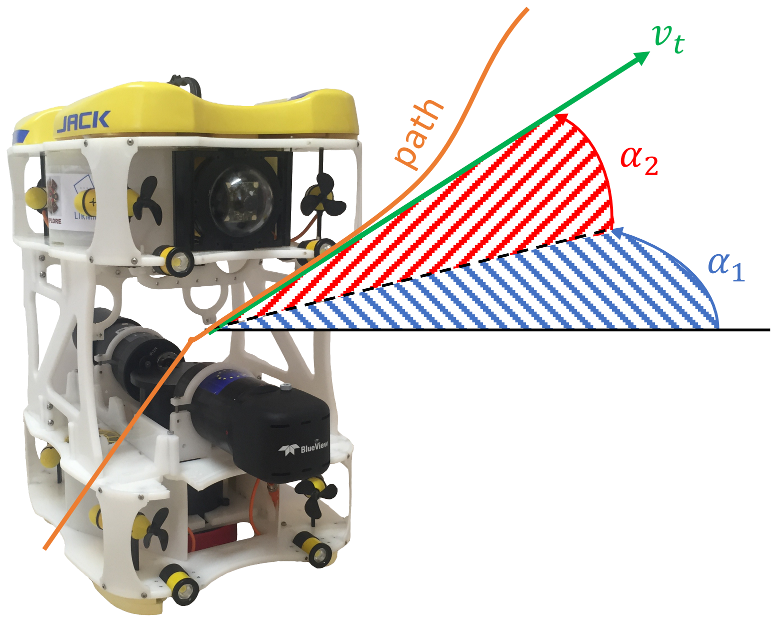
- Dead-zone effects compensation
- Robustness against motors' characteristic uncertainty and disparity
- Robustness against actuators failure : $\mathbf{r}_\mathrm{m}$? / $F_{l=1,\dots,k} = 0$ & $\mathbf{F}_\mathrm{B} = [F_1, \dots, F_m]^\mathrm{T} = \mathbf{F}_\mathrm{B}^\mathrm{d}$
  - Maximum failure ?
    - If omni-directionality preserved :
      - $\mathrm{rank}\,(\mathbf{A}_*) = 6, \mathbf{A}_* = \mathbf{A} - \{\mathbf{a}_{l=1,\dots,k}\}$

$$\underset{(6\times1)}{\mathbf{F}_\mathrm{B}} = \underset{(6\times12)}{\mathbf{A}} \cdot \underset{(12\times1)}{\mathbf{F}_\mathrm{m}}$$

$$\underset{(12\times1)}{\mathbf{F}_\mathrm{m}} = \underbrace{[\mathbf{A}^+ \quad \mathbf{M}_\mathrm{m}]}_{\substack{\mathbf{D} \\ (12\times12)}} \cdot \underset{(12\times1)}{\begin{bmatrix} \mathbf{F}_\mathrm{B}^\mathrm{d} \\ \mathbf{r}_\mathrm{m} \end{bmatrix}} \; (6\times1)$$

Redundancy

- Dead-zone effects compensation
- Robustness against motors' characteristic uncertainty and disparity
- Robustness against actuators failure : $\mathbf{r}_{\mathrm{m}}$? / $F_{l=1,\ldots,k} = 0$ & $\mathbf{F}_{\mathrm{B}} = [F_1, \ldots, F_m]^{\mathrm{T}} = \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$
  - Maximum failure ?
    - If omni-directionality preserved :
      - $\text{rank}\,(\mathbf{A}_*) = 6, \mathbf{A}_* = \mathbf{A} - \{\mathbf{a}_{l=1,\ldots,k}\}$
    - If displacement capability preserved :
      - $\text{rank}\,(\mathbf{A}_*) = 3, \mathbf{A}_* = \mathbf{A} - \{\mathbf{a}_{l=1,\ldots,k}\}$
      - Guidance & Control adaptation

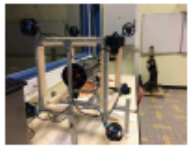


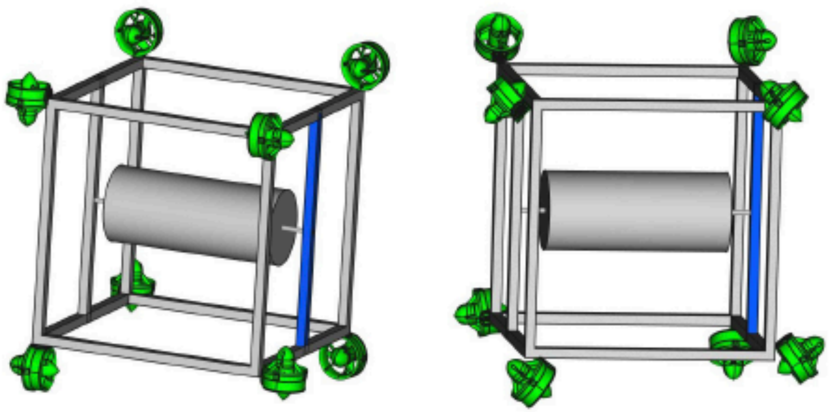
Redundancy

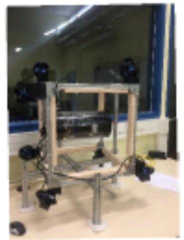
- Dead-zone effects compensation
- Robustness against motors' characteristic uncertainty and disparity
- Robustness against actuators failure : $\mathbf{r}_\mathrm{m}$? / $F_{l=1,\dots,k} = 0$ & $\mathbf{F}_\mathrm{B} = [F_1, \dots, F_m]^\mathrm{T} = \mathbf{F}_\mathrm{B}^\mathrm{d}$
  - Maximum failure ?
    - If omni-directionality preserved :
      - $\mathrm{rank}\,(\mathbf{A}_*) = 6, \mathbf{A}_* = \mathbf{A} - \{\mathbf{a}_{l=1,\dots,k}\}$
    - If displacement capability preserved :
      - $\mathrm{rank}\,(\mathbf{A}_*) = 3, \mathbf{A}_* = \mathbf{A} - \{\mathbf{a}_{l=1,\dots,k}\}$
      - Guidance & Control adaptation
- Reducing actuation activity
  - Saturation avoidance

Redundancy

- Next time :
  - Optimimal redundant design (properties of A)
    - Manipulability
    - Energy
    - Workspace
    - Reactivity
    - Robusness

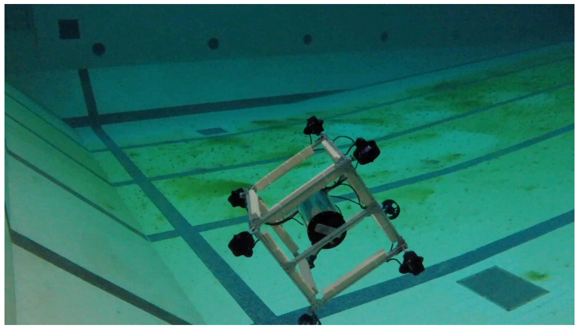






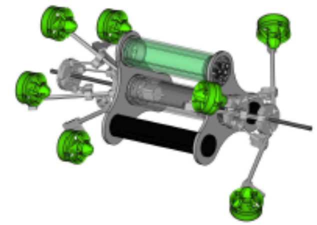
(a) Cube robot in $\mathbf{C}^1$ configuration (b) Cube robot in $\mathbf{C}^2$ configuration

Figure 19: Cube robot in two configurations $\mathbf{C}^1$ and $\mathbf{C}^2$



Redundancy

- Next time :
  - Optimimal redundant design (properties of A)
    - Manipulability
    - Energy
    - Workspace
    - Reactivity
    - Robusness
  - Dynamic management
    of variable actuation system



(a) Umbrella robot in *open-forward*

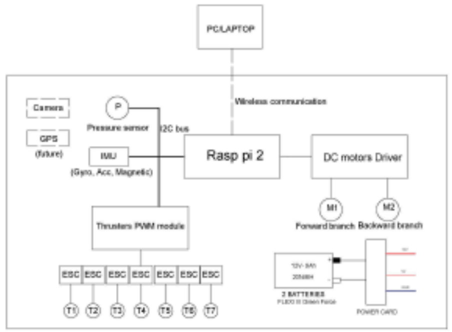
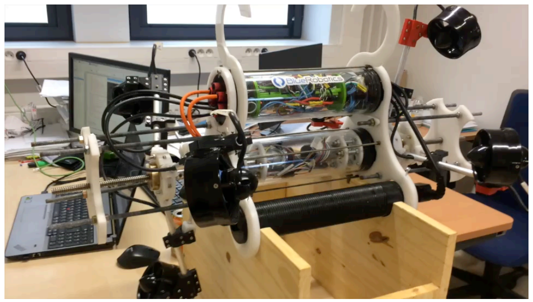
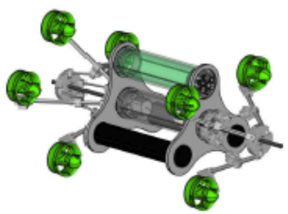
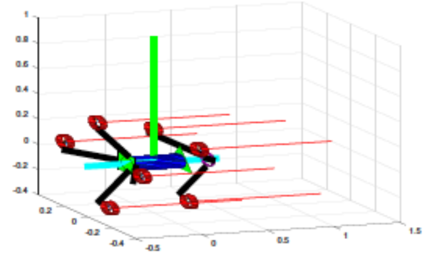


Figure 36: The principle diagram of UmRobot





(b) Umbrella robot in *close*



Redundancy

- Next time :
  - Optimimal redundant design (properties of A)
    - Manipulability
    - Energy
    - Workspace
    - Reactivity
    - Robusness
  - Dynamic management of variable actuation system
  - Final hybrid proposal