Interval-based validation of a nonlinear estimator

Maël GODARD,
Lab-STICC, ROBEX Team, ENSTA Bretagne
Tutors: Luc JAULIN (ENSTA Bretagne), Damien MASSE (UBO)

ENSTA BRETAGNE
Lab-STICC
COLLEGES SCIENCES BRETAGNE POUR L'INGENIEUR LOIRE ET LE NUMERIQUE
AGENCE INNOVATION DÉFENSE
Avec le soutien de
Région BRETAGNE

Introduction

What's an estimator ?

Examples

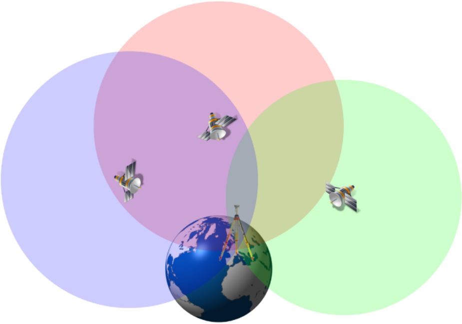


Figure: [1] An estimation : GNSS positioning

Examples

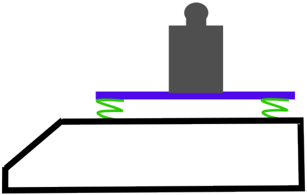


Figure: An other estimation : Weight measurement

Definitions

$\mathbf{x} \in \mathbb{X}_0$ Set of possible parameters

$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{e}$ Noisy observation

$\mathbf{e} \in [\mathbf{e}]$ Noise interval

$\hat{\mathbf{x}} = \psi(\mathbf{y})$ Estimator to validate

$\varepsilon(\mathbf{x}) = ||\mathbf{x} - \hat{\mathbf{x}}||$ Error of the estimator

$\bar{\varepsilon} = \max(\varepsilon(\mathbf{x}))$

Formalism

Equivalently, this problem can be written as below:

$$\begin{cases} \max \varepsilon(\mathbf{x}) = \|\mathbf{x} - \psi(\mathbf{g}(\mathbf{x}) + \mathbf{e})\| \\ \mathbf{x} \in \mathbb{X}_0 \\ \mathbf{e} \in [\mathbf{e}] \end{cases}$$

It can be interpreted as a maximization problem of ε or as a minimization problem of $-\varepsilon$.

Definition

An Optimization problem is defined by:

- An objective function to minimize $f : \mathbb{R}^n \mapsto \mathbb{R}$
- A domain $\mathbb{X}_0 \subseteq \mathbb{R}^n$
- A set of conditions $g_i\{x_1, \ldots, x_n\} \leq 0$ for $i \in \{1, \ldots, m\}$. g_i are functions of type $\mathbb{R}^n \mapsto \mathbb{R}$.

The Moore-Skelboe algorithm

The Moore-Skelboe algorithm gives an box containing the global minimum of a function with width inferior to a choosen criteria, noted δ below:

let the cover be $\{B_0\}$
while $(w(f(B_0)) > \delta)$ {
  // $\mu \in f(B_0)$
  remove B_0 from the cover
  split B_0 and insert the results into
  the cover in non decreasing order of
  $lb(f(B_i))$, for $i = 0, \ldots, N-1$
}
// $\mu \in f(B_0)$ and $w(f(B_0)) < \delta$
output $f(B_0)$

Figure: [2] Moore-Skelboe algorithm (MS0)

Example

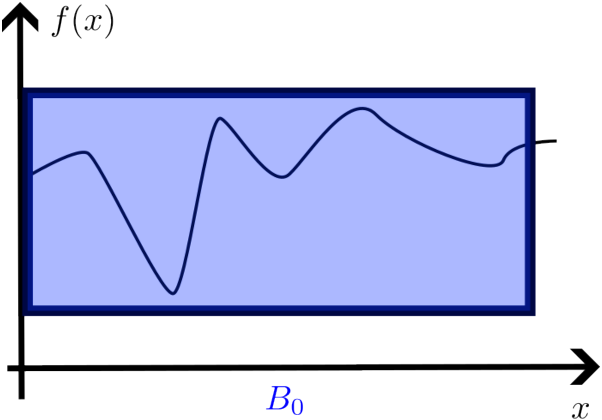


Figure: Moore-Skelboe algorithm - Step 1

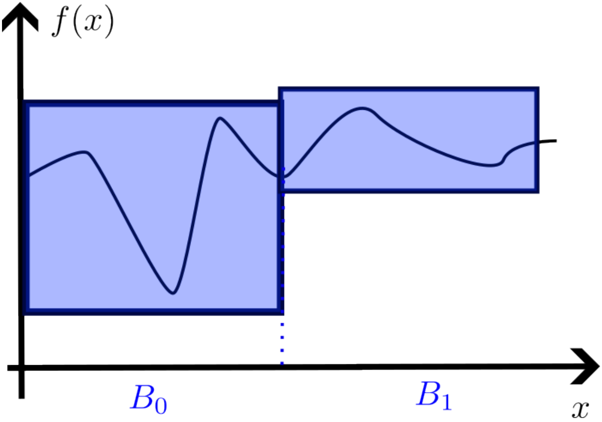


Figure: Moore-Skelboe algorithm - Step 2

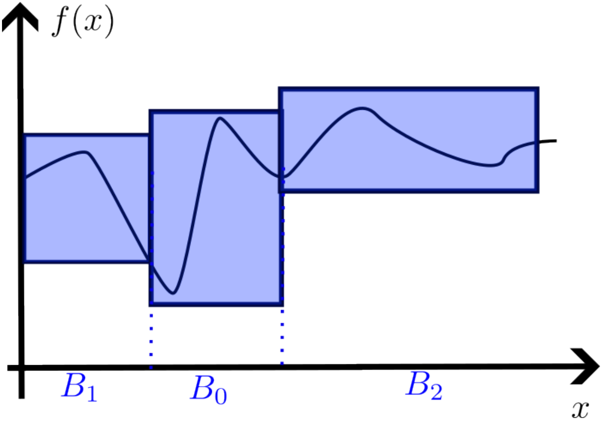


Figure: Moore-Skelboe algorithm - Step 3

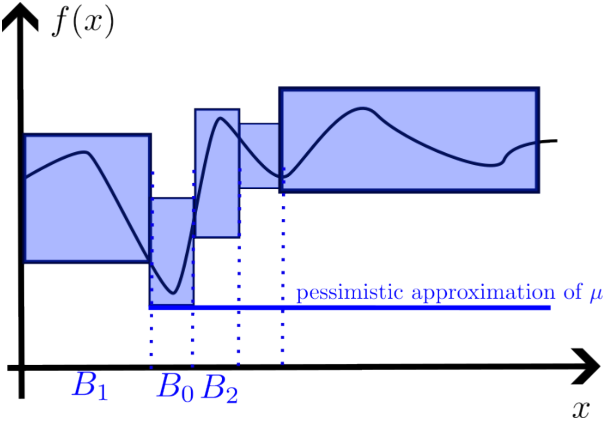
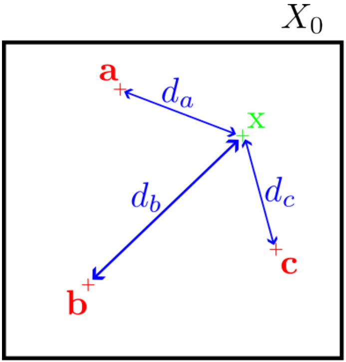


Figure: Moore-Skelboe algorithm - Step 4

Problem



$$\mathbb{X}_0 = [5, 25]^2$$

$$[\mathbf{e}] = [-0.2, 0.2]^3$$

$$a = (10, -9)$$
$$b = (5, 12)$$
$$c = (-15, 0)$$

$$\mathbf{g}(\mathbf{x}) = \begin{pmatrix} d_a \\ d_b \\ d_c \end{pmatrix}$$

Figure: Problem description

Gradient descent Estimator

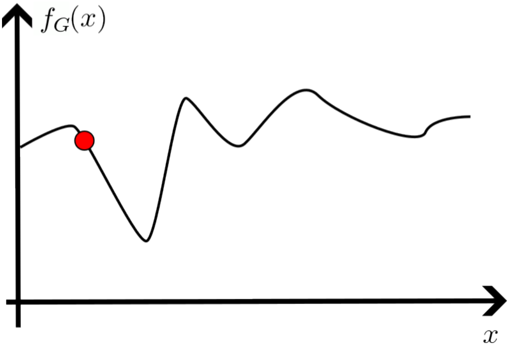


Figure: Gradient descent algorithm - Step 1

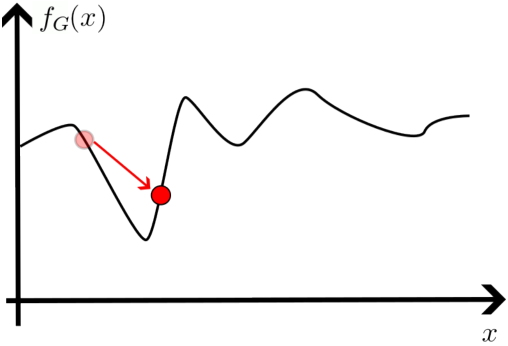


Figure: Gradient descent algorithm - Step 2

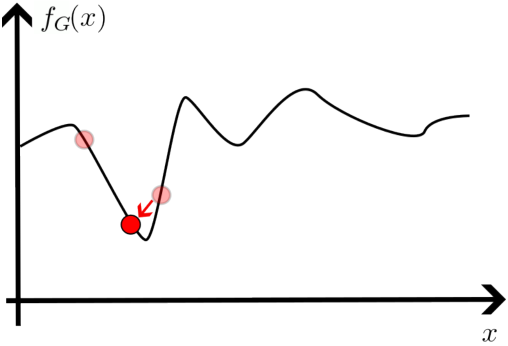
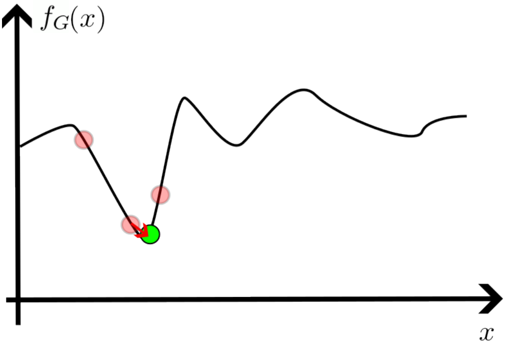


Figure: Gradient descent algorithm - Step 3



Applied to $f_G(\mathbf{x}) = \|\mathbf{y} - \mathbf{g}(\mathbf{x})\|$, $\bar{\varepsilon} = 8.4$m

Figure: Gradient descent algorithm - Step 4

Gradient descent estimator bad case

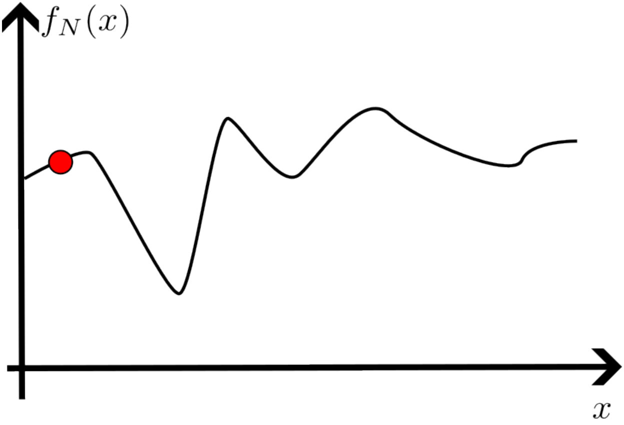


Figure: Gradient descent algorithm - Step 1

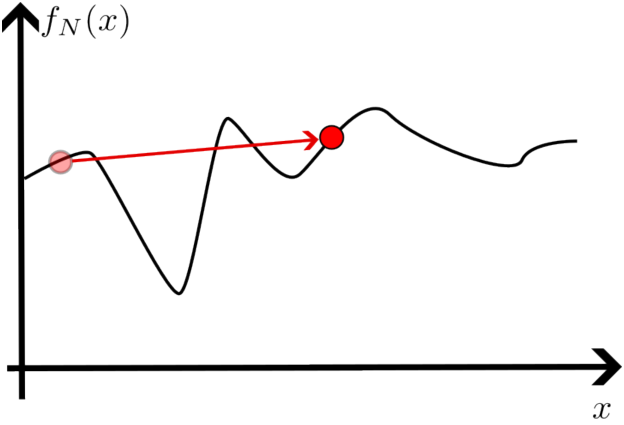


Figure: Gradient descent algorithm - Step 2

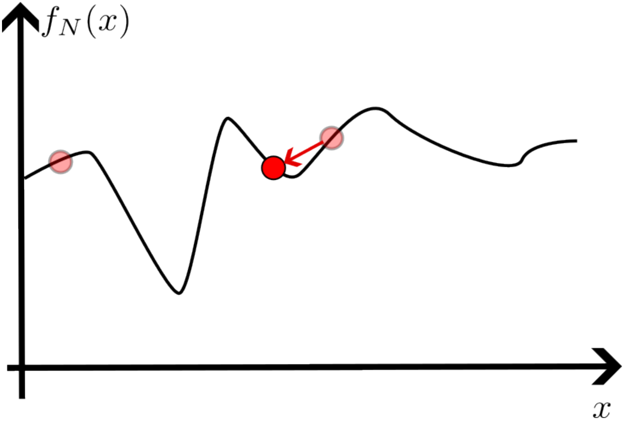


Figure: Gradient descent algorithm - Step 3

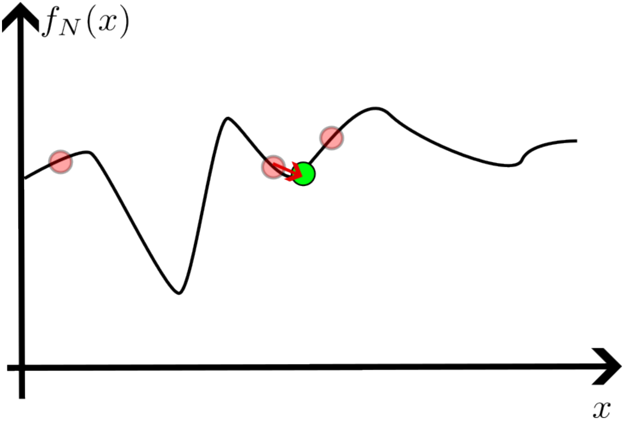


Figure: Gradient descent algorithm - Step 4

CNN Estimator

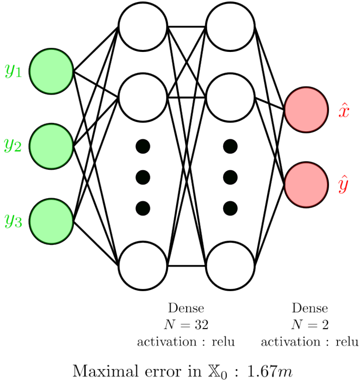


Figure: Neural Network Estimator

Simulation

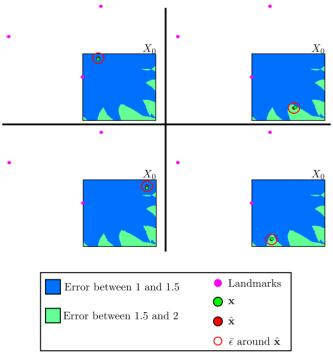


Figure: Visualization of $\bar{\varepsilon}$

Conclusion

- Validation of all nonlinear estimator (non interval-based)
- Guaranteed

Bibliography

[1] Bosser P., Support de cours, GNSS : Systèmes globaux de positionnement par satellite, 2017.

[2] van Emden M., Moa B., Termination Criteria in the Moore-Skelboe Algorithm for Global Optimization by Interval Arithmetic, 2004.

[3] Godard M., Jaulin L., Masse D., Interval-based validation of a nonlinear estimator, Under study.