Achieving stable formation control for two ROVs

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Outline

Formation control needs practical applications

- Moving in a formation makes the group more **reliable**
- many **theoretical controllers** are proposed for formation control [3, 4, 1, 2]
- There is a need to study **more complex systems** (multi-agent, underwater perturbation, communication issues,...)
- **Real case application** are still rare

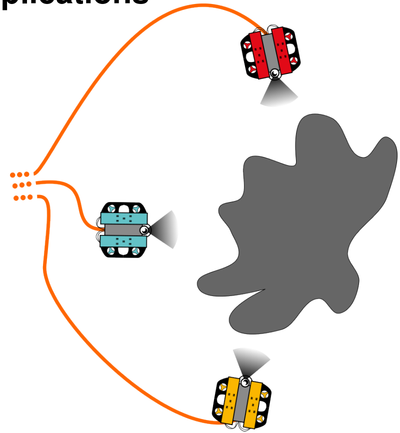

Figure: Collaborative inspection with ROVs (Remotely Operated Vehicles)

Example - acoustic localization with little information

- Global position measured by USBL (Ultra Short Base-Line)
- Position is measured every 6s
- **Can we achieve this formation without dead reckoning?**

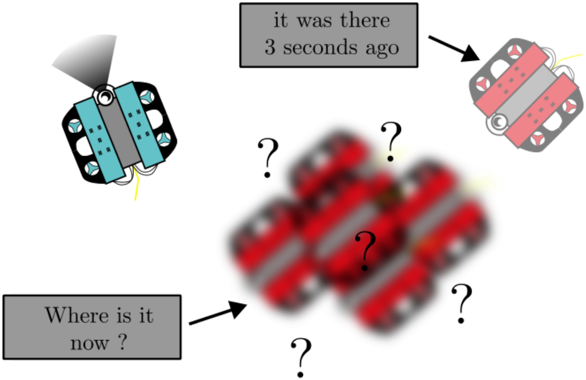

Figure: The information on positions is limited

Can we achieve this formation without dead reckoning ?

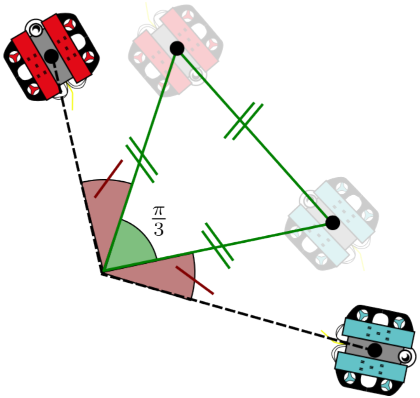

Figure: Equilateral triangle formation with Virtual Structure and pose tracking

Proof of stability for ROVs formation control

Modelling a synchronous hybrid system with Two ROVs (1/7)

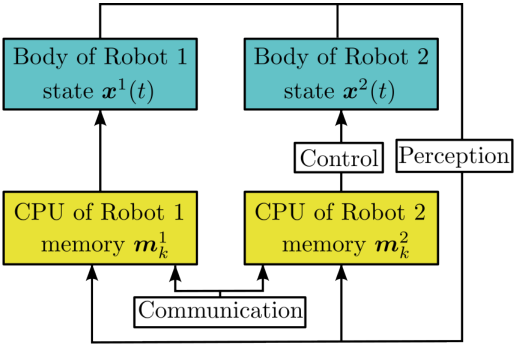

Figure: Cyber-Physical multi-agent system, a synchronous hybrid system

Modelling a synchronous hybrid system with Two ROVs (2/7)

Horizontal positions

$$\boldsymbol{p}_b = d_b \cdot \begin{bmatrix} \cos(\phi_b) \\ \sin(\phi_b) \end{bmatrix} \in \mathbb{R}^2 \quad (1)$$

$$\boldsymbol{p}_r = d_r \cdot \begin{bmatrix} \cos(\phi_r) \\ \sin(\phi_r) \end{bmatrix} \in \mathbb{R}^2 \quad (2)$$

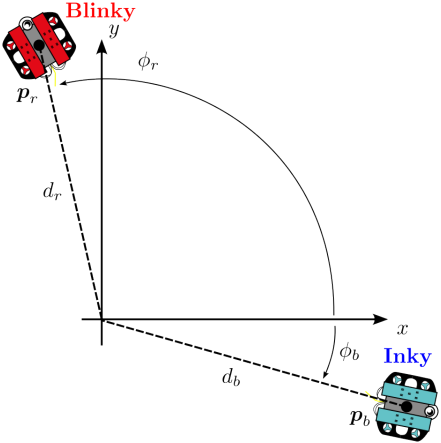

Figure: Cartesian and polar coordinates

Modelling a synchronous hybrid system with Two ROVs (3/7)

- Time of the discrete update (control and measurement) $t_k = k \cdot T_a$ with the period $T_a > 0$.
- ROV are modeled as simple integrators. For $t \in [t_k, t_{k+1}[$,

$$\begin{aligned} \dot{\mathbf{p}}_b(t) &= \mathbf{u}_{b,k} \\ \dot{\mathbf{p}}_r(t) &= \mathbf{u}_{r,k} \end{aligned} \tag{3}$$

- Let $i \in \mathbb{N}$. Measuring for *Inky* at time t_{2i} and for *Blinky* at time t_{2i+1}. The position memory

$$\begin{aligned} \mathbf{m}_{b,2i} = \mathbf{m}_{b,2i+1} &= \mathbf{p}_b(t_{2i}) \\ \mathbf{m}_{r,2i+1} = \mathbf{m}_{r,2i+2} &= \mathbf{p}_r(t_{2i+1}) \end{aligned} \tag{4}$$

Modelling a synchronous hybrid system with Two ROVs (4/7)

Desired positions:

$$\mathbf{p}^*_{b,k} = d^* \cdot \begin{bmatrix} \cos\left(\phi_k - \frac{\pi}{6}\right) \\ \sin\left(\phi_k - \frac{\pi}{6}\right) \end{bmatrix}$$
$$\mathbf{p}^*_{r,k} = d^* \cdot \begin{bmatrix} \cos\left(\phi_k + \frac{\pi}{6}\right) \\ \sin\left(\phi_k + \frac{\pi}{6}\right) \end{bmatrix} \quad (5)$$

with the desired distance $d^* > 0$ and the orientation of the triangle ϕ_k given by

$$\phi_k = \frac{\arg\left(\mathbf{m}_{r,k}\right) + \arg\left(\mathbf{m}_{b,k}\right)}{2}. \quad (6)$$

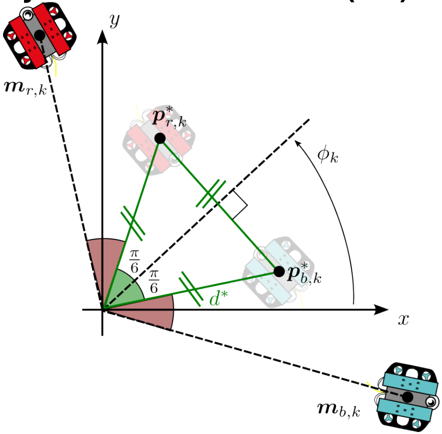

Figure: Equilateral triangle formation with Virtual Structure and pose tracking

Modelling a synchronous hybrid system with Two ROVs (5/7)

Proportional controller, with the gain $k_p > 0$:

$$\begin{aligned} \boldsymbol{u}_{b,k} &= k_p \cdot \left(\boldsymbol{p}^*_{b,k} - \boldsymbol{m}_{b,k}\right) \\ \boldsymbol{u}_{r,k} &= k_p \cdot \left(\boldsymbol{p}^*_{r,k} - \boldsymbol{m}_{r,k}\right) \end{aligned} \qquad (7)$$

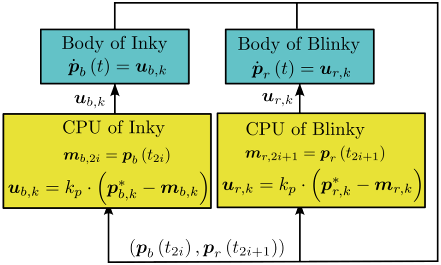

Figure: Cyber-Physical multi-agent system, a synchronous hybrid system

Modelling a synchronous hybrid system with Two ROVs (6/7)

- Global state vector $\boldsymbol{z} \in \mathbb{R}^8$
- z_1, z_2 and z_3 are continuous
- z_4, z_5 and z_6 are discrete
- z_7 and z_8 are piece-wise continuous
- Equilibrium point $\boldsymbol{z}_{eq} = 0$
- Periodic discrete evolution

$$\boldsymbol{z}_{2i+2} = \boldsymbol{h}(\boldsymbol{z}_{2i}) \quad (8)$$

$$\boldsymbol{h} = \phi_T \circ \boldsymbol{g}_r \circ \phi_T \circ \boldsymbol{g}_b \quad (9)$$

with the continuous-time evolution ϕ_T and the discrete updates $\boldsymbol{g}_r$ and $\boldsymbol{g}_b$.

Modelling a synchronous hybrid system with Two ROVs (7/7)

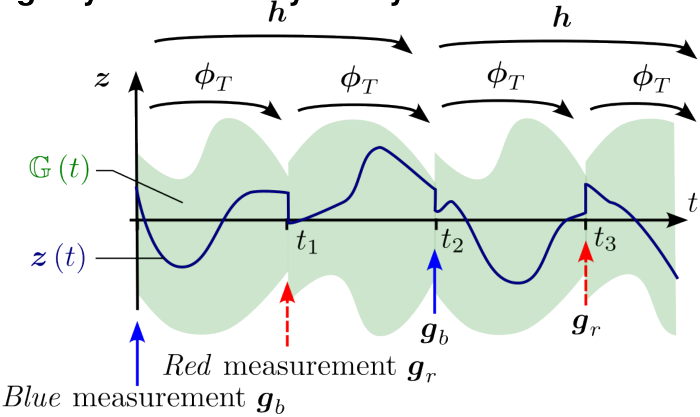

Figure: Time evolution of the global state. $\boldsymbol{h} = \phi_T \circ \boldsymbol{g}_r \circ \phi_T \circ \boldsymbol{g}_b$. The state stay in the Tube $\mathbb{G}(t)$

The effect of the controller gain, in Simulation

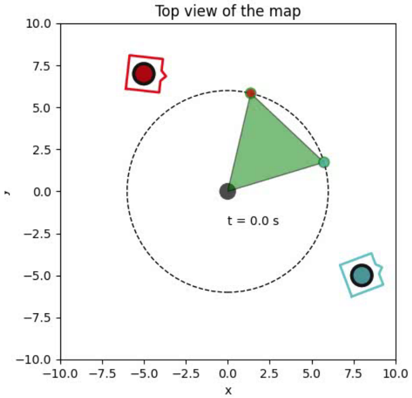

Figure: Simulation with $k_p = 0.1$

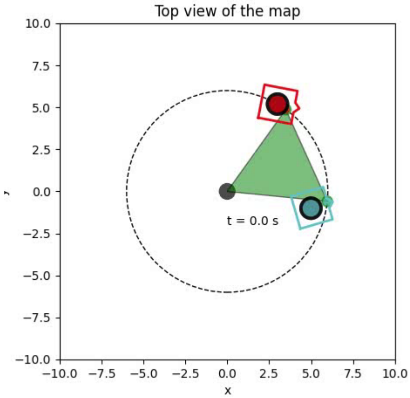

Figure: Simulation with $k_p = 1$

How ???

How do we prove the stability of this system ???

Stability Theory

- We can study the discrete system

$$\begin{aligned} \boldsymbol{z}_{2i+2} &= \boldsymbol{h}(\boldsymbol{z}_{2i}), \\ 0 &= \boldsymbol{h}(0). \end{aligned} \tag{10}$$

- But we don't have the analytical expression of $\boldsymbol{h}$.
- So, we can't compute the eigenvalues of the **Jacobian Matrix** of $\boldsymbol{h}$.
- And, we can't use **Lyapunov functions**.

A possible solution: use **guaranteed integration** to prove the existence of **positive invariant ellipsoids**.

Stability analysis with positive invariant ellipsoid

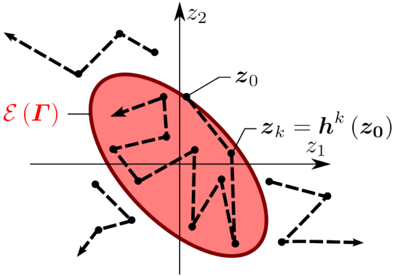

Figure: The state cannot escape a PI (Positive invariant) ellipsoid

Definition of a non-degenerated ellipsoid

$$\mathcal{E}(\boldsymbol{\Gamma}) = \left\{ \boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{x}^T \boldsymbol{\Gamma}^{-T} \boldsymbol{\Gamma}^{-1} \boldsymbol{x} \leq 1 \right\} \quad (11)$$

with $\boldsymbol{\Gamma} \in \mathbb{R}^{n \times n}$ and the positive definite shape matrix $\boldsymbol{\Gamma}\boldsymbol{\Gamma}^T \in S_n^+$.

Stability analysis with positive invariant ellipsoid

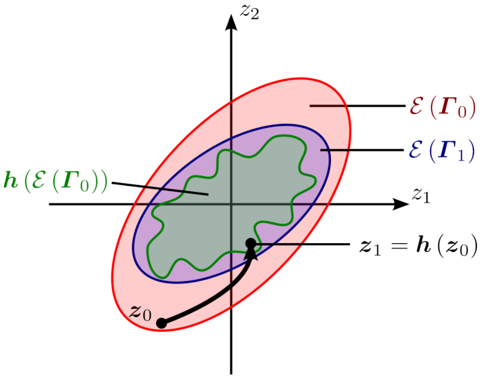

Figure: Illustration of the Method

Method to prove the existence of a PI ellipsoid

1. Choose a candidate $\mathcal{E}(\boldsymbol{\Gamma}_0)$
2. With guaranteed integration, Compute an enclosing ellipsoid $\mathcal{E}(\boldsymbol{\Gamma}_1)$, such that $\boldsymbol{h}(\mathcal{E}(\boldsymbol{\Gamma}_0)) \subseteq \mathcal{E}(\boldsymbol{\Gamma}_1)$.
3. Verify the inclusion $\mathcal{E}(\boldsymbol{\Gamma}_1) \subseteq \mathcal{E}(\boldsymbol{\Gamma}_0)$ to prove that $\mathcal{E}(\boldsymbol{\Gamma}_0)$ is positive invariant.

Computation time

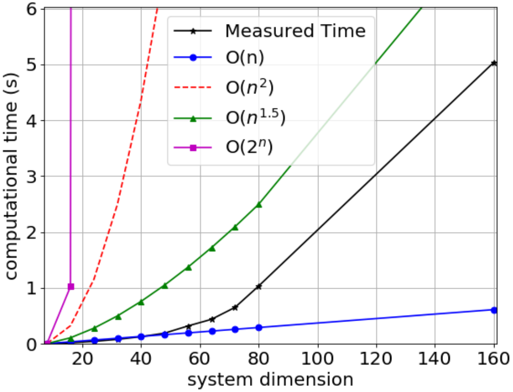

Figure: This numerical method has a polynomial complexity

Illustration of a 8-dimensional ellipsoid

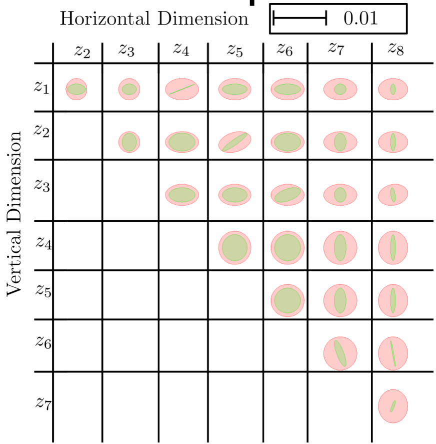

Figure: Projections of the ellipsoids $\mathcal{E}(\Gamma_0)$ (red) and $\mathcal{E}(\Gamma_1)$ (green)

Presentation of the Real system (1/2)

Figure: X150 Mirco-USBL USBL fixed on a pole

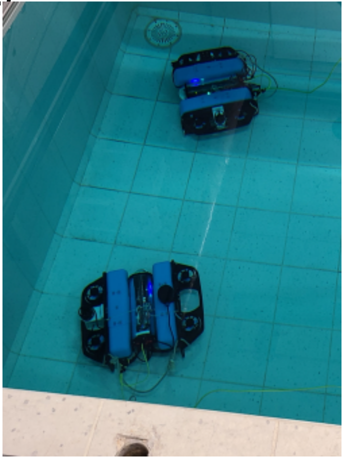

Figure: BlueROV2 *Inky* and *Blinky* in the Pool of the ENSTA Bretagne

Presentation of the Real system (2/2)

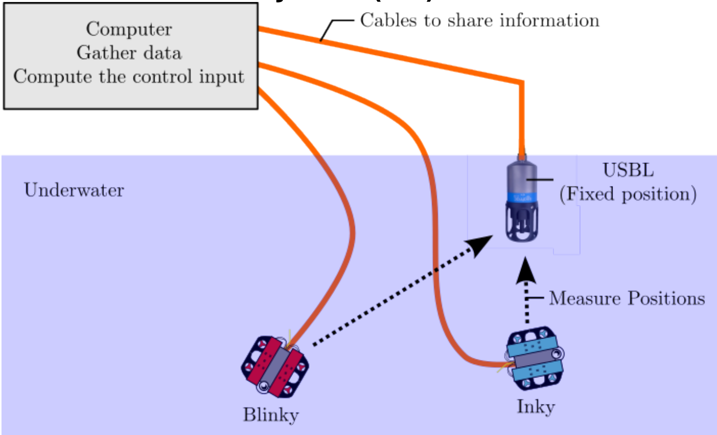

Figure: Architecture of the system

Context of the Experiment

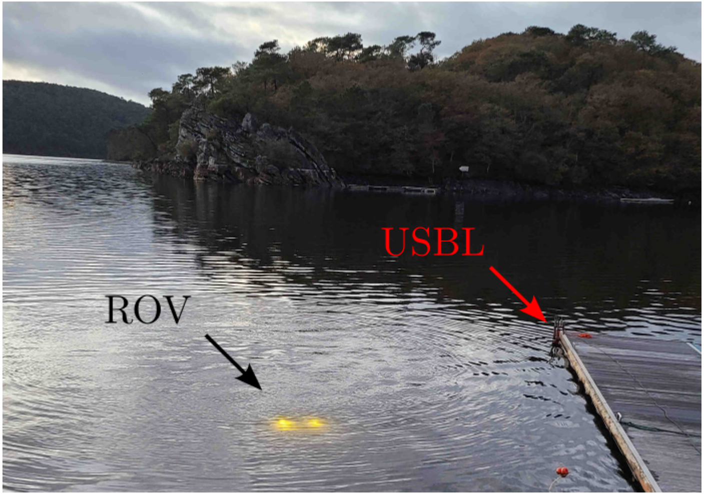

Figure: Experiment at the lake of Guerledan

Reconstruction of the Experiment

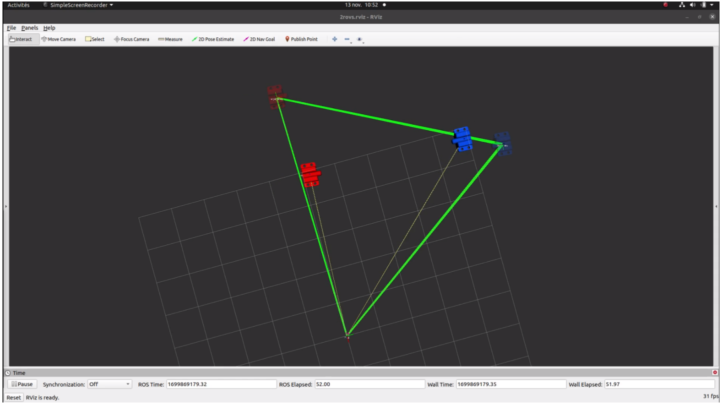

Figure: Display of the Data on Rviz

Evolution of the memory state

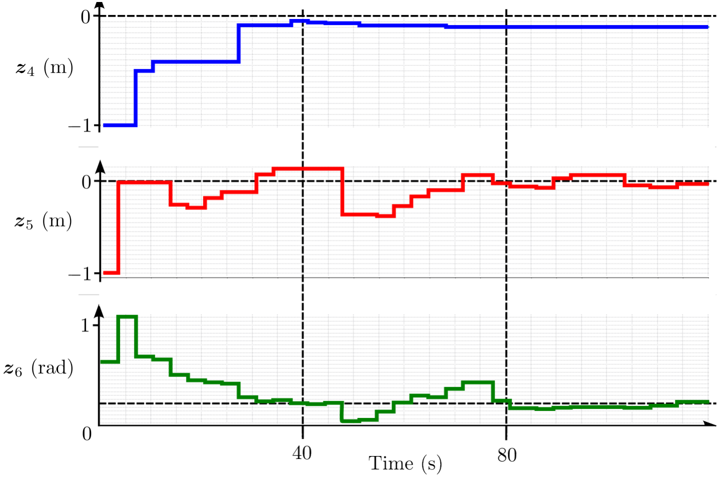

Figure: Evolution of z_4, z_5 and z_6 during this experiment

Conclusion

Results:

- We proved the stability of a **synchronous nonlinear hybrid system**
- We achieved correct formation control in practice

Future study:

- Find bigger positive invariant ellipsoids
- Enhance the localization of the ROVs (tune the USBL, predict movement based on propeller input, identify the mechanical model of the ROV,...)

Bibliography I

- Camila M. G. Gussen, Paulo S. R. Diniz, Marcello L. R. Campos, Wallace A. Martins, Felipe M. Costa, and Jonathan N. Gois.
  A Survey of Underwater Wireless Communication Technologies.
  Journal of Communication and Information Systems, 31, October 2016.
  Number: 1.
- Kwang-Kyo Oh, Myoung-Chul Park, and Hyo-Sung Ahn.
  A survey of multi-agent formation control.
  Automatica, 53:424–440, March 2015.
- Auwal Shehu Tijjani, Ahmed Chemori, and Vincent Creuze.
  A survey on tracking control of unmanned underwater vehicles: Experiments-based approach.
  Annual Reviews in Control, September 2022.

Bibliography II

Yue Yang, Yang Xiao, and Tieshan Li.
A Survey of Autonomous Underwater Vehicle Formation: Performance, Formation Control, and Communication Capability.
IEEE Communications Surveys & Tutorials, 23(2):815–841, 2021.
Conference Name: IEEE Communications Surveys & Tutorials.