Stable cycles for Autonomous Underwater Vehicles navigation

Quentin Brateau, Fabrice Le Bars, Luc Jaulin

December 13, 2024

ENSTA Bretagne

Context

Research laboratory

- ENSTA Bretagne, UMR 6285, Lab-STICC, IAO, ROBEX

Supervisiors

- Luc Jaulin
- Fabrice Le Bars

Funding

- AID funding: Jean-Daniel Masson

AUV

- Control of torpedo-like AUV
- Riptide's micro-uuv

Environment

- Constrained environment
- Pool, harbor, ...

Goals

- Reactivity
- Manoeuvrability

Figure 1: Harbor and Riptide in the ENSTA Bretagne pool

Introduction

Figure 2: Earth-orbiting positioning and communications satellite

Figure 3: Riptide's last memory [a]

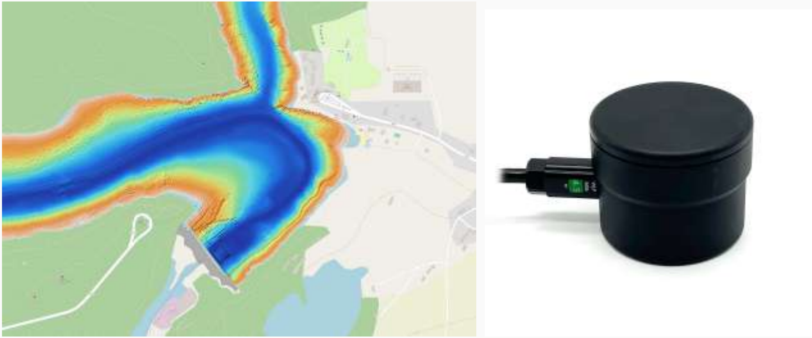
[a] https://www.youtube.com/watch?v=hNqIShmMQjA

Metric vs Topological map

(a) Metric map

(b) Topological map - Saint-Sèvre

Figure 4: World metric and topological map



(a) Guerlédan bathymetric map (b) Ping2 Blureobotics [1]

Figure 5: Digital elevation model and depth sensing

[1] https://bluerobotics.com/

Dynamical system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \tag{1}$$

Flow Function

$$\begin{aligned} &\forall(\mathbf{x}_0, t1, t2) \in \mathcal{S} \times \mathcal{T}^2 \\ &\boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t_1) = \mathbf{x}(t_1) \\ &\boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, 0) = \mathbf{x}_0 \\ &\boldsymbol{\varphi}_{\mathbf{u}}(\boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t1), t2) = \boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{x}_0, t1 + t2) \end{aligned} \tag{2}$$

Dubins car

$$\mathbf{f}(\mathbf{x}, u) = \begin{bmatrix} V \cdot \cos(\theta) \\ V \cdot \sin(\theta) \\ u \end{bmatrix} \tag{3}$$

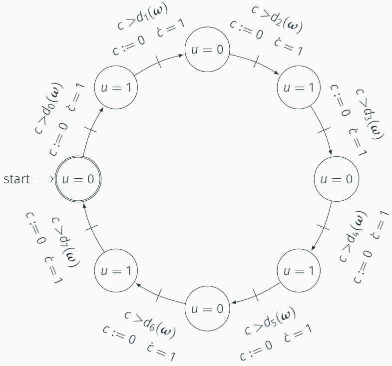


Figure 6: Square cyclic timed automata

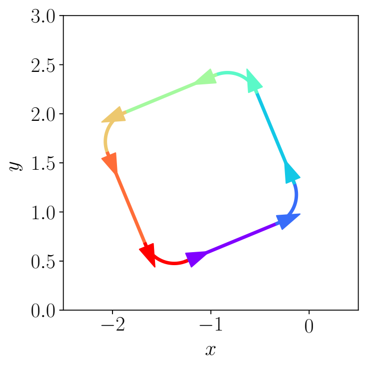
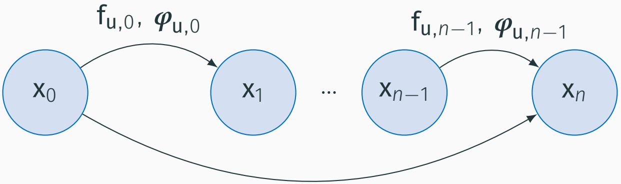


Figure 7: Square Cycle



$$\boldsymbol{\phi}(\mathbf{x}_0) = \bigcirc_{i=0}^{n-1} \boldsymbol{\varphi}_{u,i}(\mathbf{x}_0, d_i(\boldsymbol{\omega}))$$

Figure 8: Composition of flow functions over a cycle

Cyclic Period

$$T(\boldsymbol{\omega}) = \sum_{i=0}^{n-1} d_i(\boldsymbol{\omega}) \tag{4}$$

Synchronization Condition

$$\boldsymbol{\phi}(\mathbf{x}(t)) \triangleq \mathbf{x}(t + T(\boldsymbol{\omega})) = \mathbf{x}(t) \tag{5}$$

Cycle abstraction and Shapes

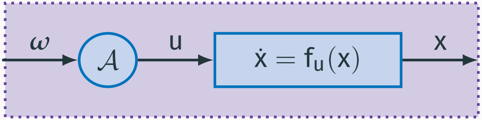


Figure 9: Block diagram of the robot controlled by a timed automata

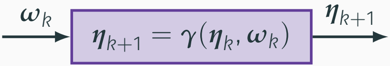


Figure 10: Block diagram of the controlled cycle

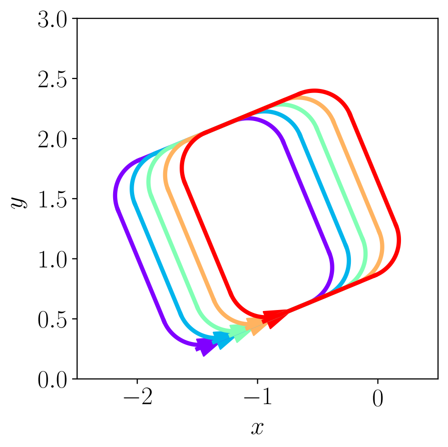


Figure 11: Cycle move with $\omega_0 = -0.15$

Cycle move through iterations

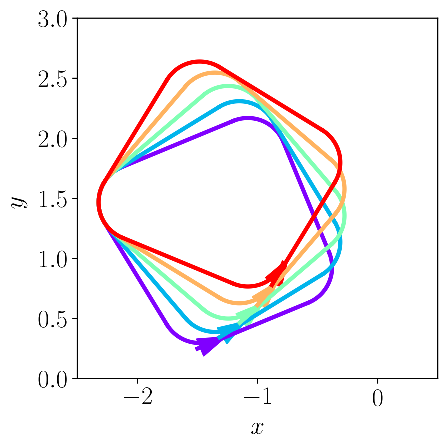


Figure 11: Cycle move with $\omega_1 = 0.1$

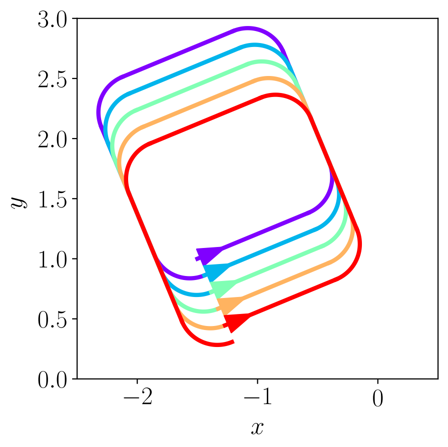


Figure 11: Cycle move with $\omega_2 = 0.15$

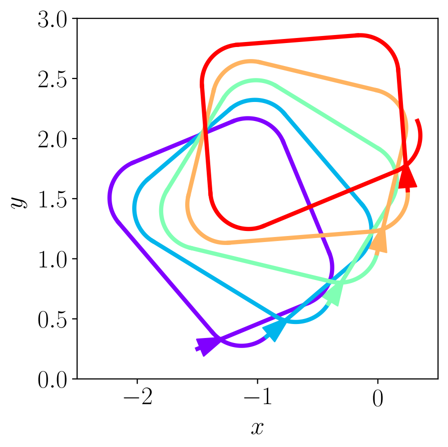


Figure 11: Control of the cycle with $\boldsymbol{\omega} = \begin{bmatrix} -0.1 & 0.2 & 0.2 \end{bmatrix}^T$

Feedforward linearization

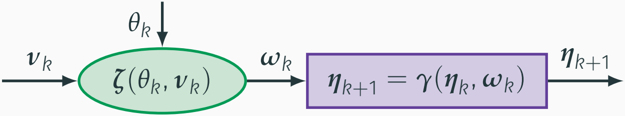


Figure 12: Block diagram of the controlled cycle

$$\zeta(\theta_k, \nu_k) = \begin{bmatrix} -cos(\theta_k) & 0 & sin(\theta_k) \\ -sin(\theta_k) & 0 & -cos(\theta_k) \\ 0 & 1 & 0 \end{bmatrix} \cdot \nu_k \qquad (6)$$

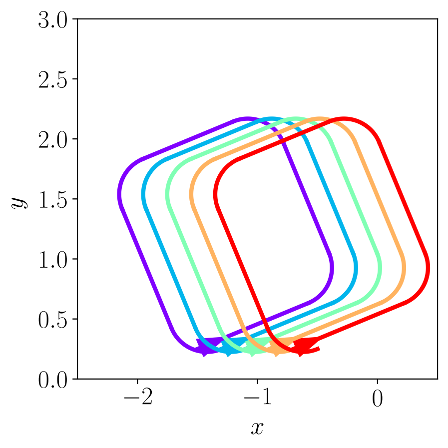


Figure 13: Cycle move with $\nu_0 = 0.2$

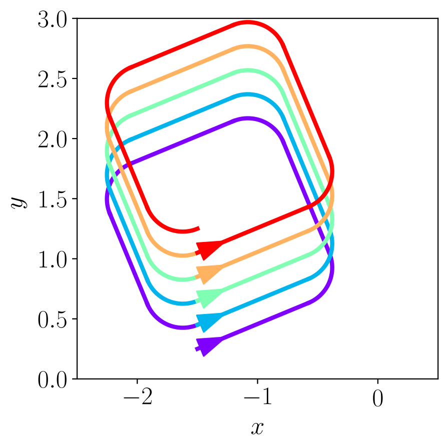


Figure 13: Cycle move with $\nu_1 = 0.2$

Cycle world move through iterations

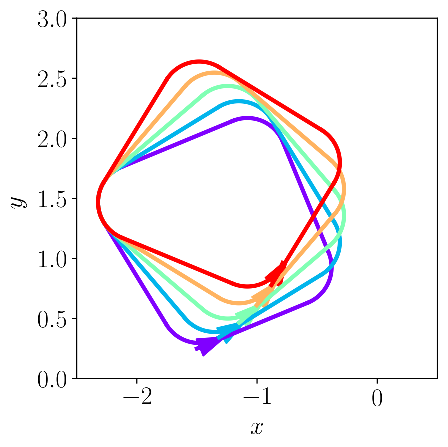
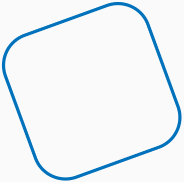


Figure 13: Cycle move with $\nu_2 = 0.1$



Shape concept

Discrete system movable in the 2D plane through iterations.

Shape action

- Translation
- Rotation
- Homotethy

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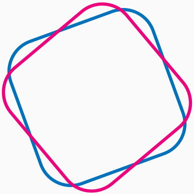

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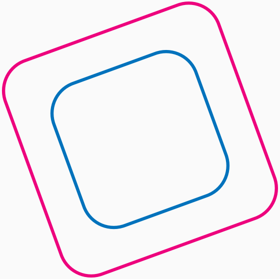

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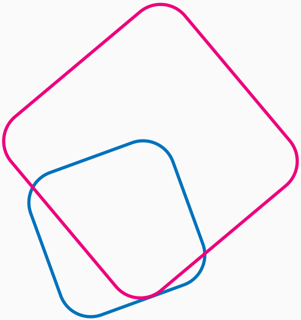

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Adding measurements

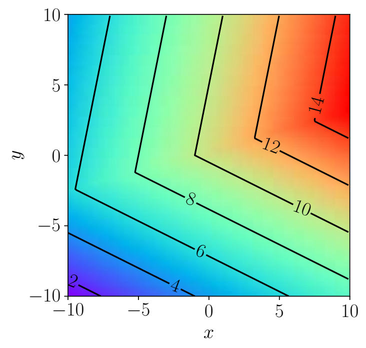

Figure 14: Bathymetric map

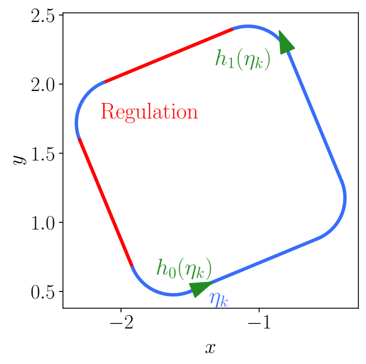

Figure 15: Measurements position

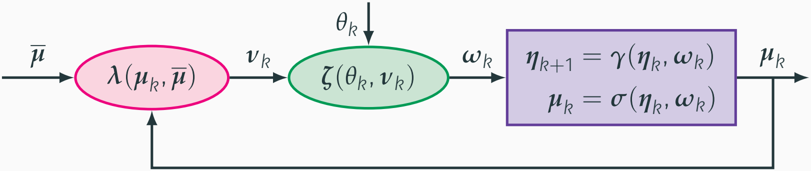

Figure 16: Block diagram of the autonomous system

$$\lambda(\mu_k, \overline{\mu}) = K \cdot \arctan\left(\frac{\overline{\mu} - \mu_k}{r}\right) \tag{7}$$

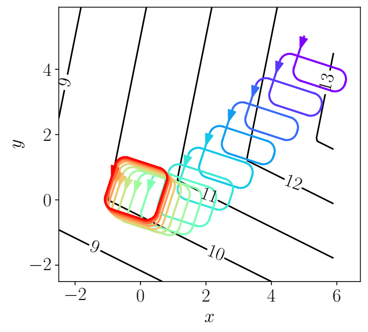

Figure 17: Measurements simulation

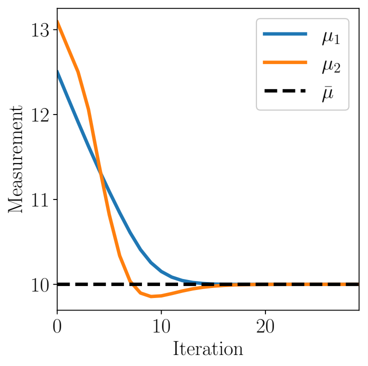

Figure 18: Measurements results

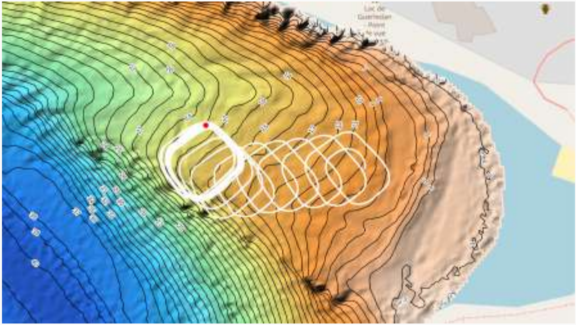

Figure 19: Guerlédan's Lake Stable Cycle Trial [a]

[a] https://www.youtube.com/watch?v=MDJ6iHYhxyM

Cycle navigation

Cycle element

Given a topological space X, $\pi(X, x_0)$ is the pointed space at x_0

$$\forall \gamma \in \pi(X, x_0), \begin{cases} \gamma : [0,1] \to X \\ \gamma(0) = \gamma(1) = x_0 \end{cases} \tag{8}$$

Figure 20: Cycles $(\gamma_0, \gamma_1) \in \pi(X, x_0)$

Concatenation and Cycle group

$\forall (\gamma_0, \gamma_1) \in \pi(X, x_0)^2$,

$$\gamma_0 \cdot \gamma_1 = \begin{cases} \gamma_0(t), & 0 \leq t < \frac{1}{2} \\ \gamma_1(t), & \frac{1}{2} \leq t \leq 1 \end{cases} \tag{9}$$

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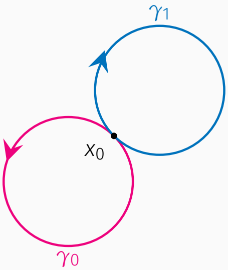

Figure 20: Cycles $(\gamma_0, \gamma_1) \in \pi(X, x_0)$

Cycle switch

- Start from a stabilized cycle
- Reach the capture basin of the next cycle

Relationship between cycles

- Define a binary relationship operator $\mathcal{R}$ between elements of cycle group

$\mathcal{R}$ is a Preorder

- **Reflexivity**: $\gamma_0 \mathcal{R} \gamma_0$
- **Transitivity**: $\gamma_0 \mathcal{R} \gamma_1 \wedge \gamma_1 \mathcal{R} \gamma_2 \implies \gamma_0 \mathcal{R} \gamma_2$

Cycle switch

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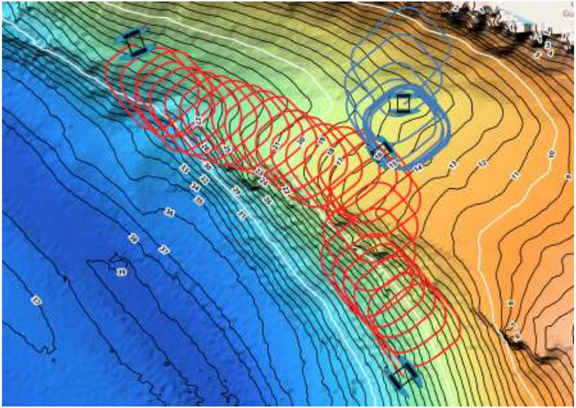

Figure 21: Two trajectories - The blue one starts in the capture basin of the cycle, the red one not.

Reflexivity

$\forall i \in [\![0,4]\!], r_i = id$

Transitivity

$\forall i \in [\![4,11]\!], r_i$

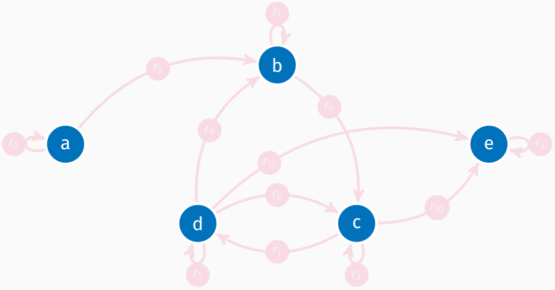

Figure 22: Graph of $\mathcal{R}$ relationship

Reflexivity

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Transitivity

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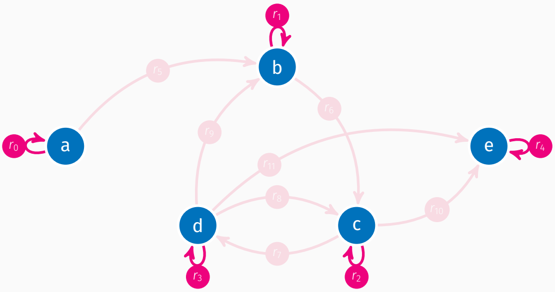

Figure 22: Graph of $\mathcal{R}$ relationship

Cycle navigation example

Reflexivity
$\forall i \in [\![0, 4]\!], r_i = id$

Transitivity
$\forall i \in [\![4, 11]\!], r_i$

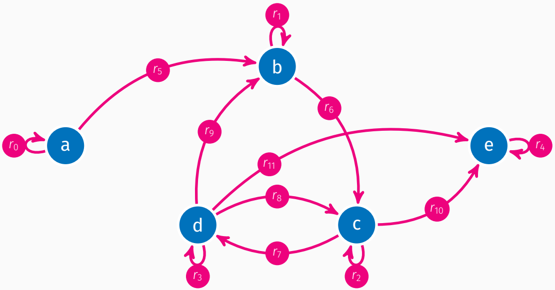

Figure 22: Graph of $\mathcal{R}$ relationship

Notable cycles

Starting cycles

- Unable to reach once leave
- Let robot stabilize before starting

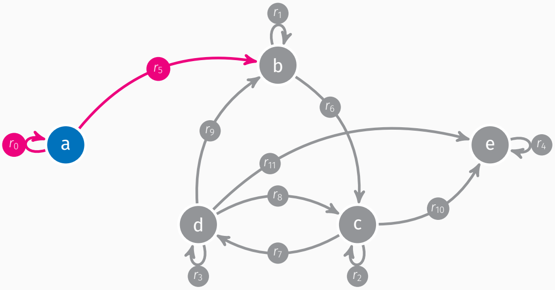

Figure 23: Starting Cycles

Navigating cycles

- Used for navigation
- $r_8 = r_9 \cdot r_6$

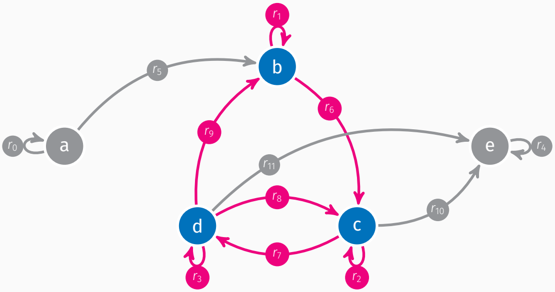

Figure 23: Navigating Cycles

Recovering cycles

- Unable to leave once reached
- Let system be recovered at known location

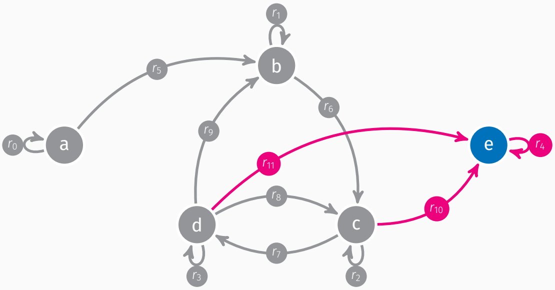

Figure 23: Recovering Cycles

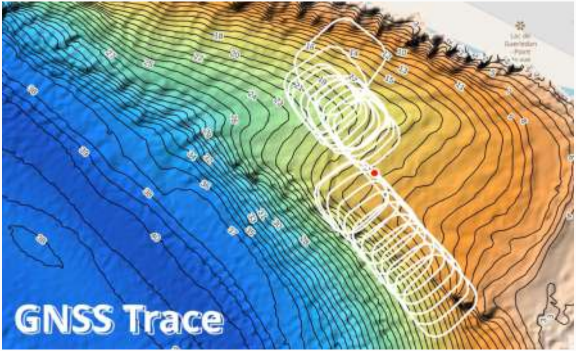

Figure 24: Guerlédan's Lake Cycle Switch Trial [a]

[a] https://www.youtube.com/watch?v=MDJ6iHYhxyM

Cycle navigation and Worlds

Strongly connected graph

There is a path in each direction between each pair of vertices of the graph.

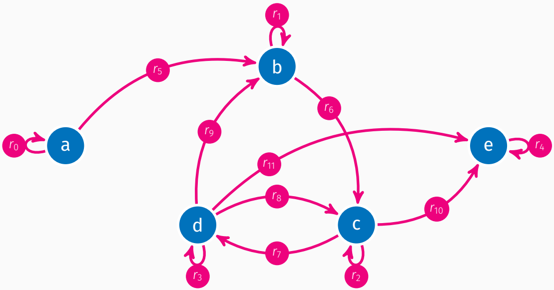

Figure 25: Not strongly connected graph

Strongly connected graph

There is a path in each direction between each pair of vertices of the graph.

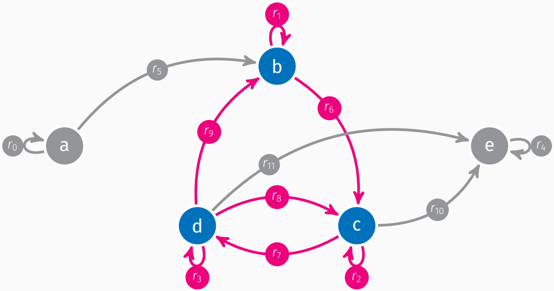

Figure 25: Strongly connected graph

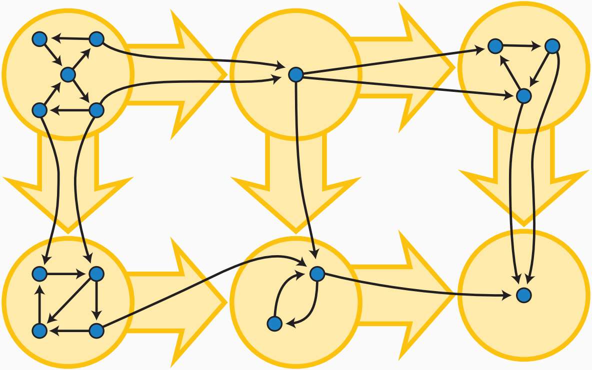

Figure 26: Strongly connected components graph decomposition - World concept

Conclusion

Contributions

- Introduction of stable cycles
- Concept of Cycle Navigation
- Validation of this concept in simulation and real environment

Perspectives

- Test with underwater robots
- Complex mission involving navigation through Worlds

Questions?

Stable cycles for Autonomous Underwater Vehicles navigation

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