Loop detection with proprioceptive sensors

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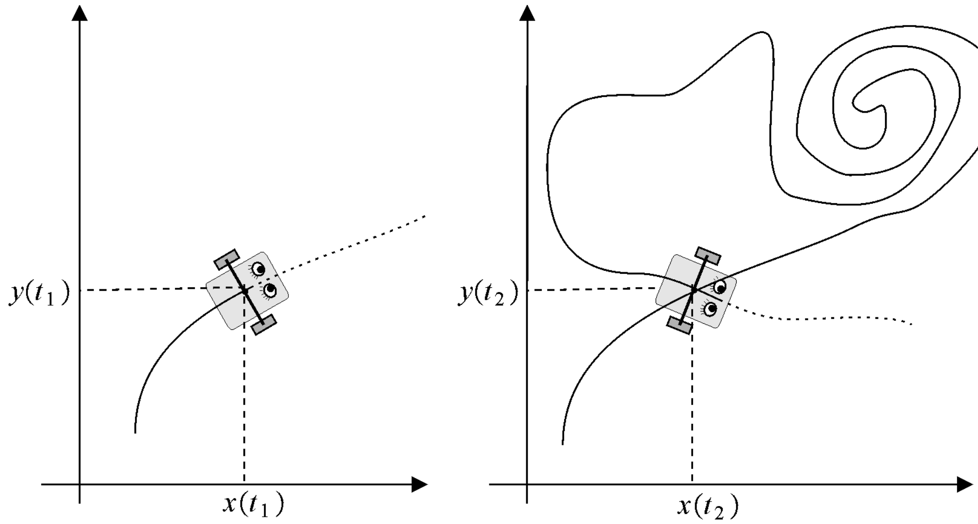
1 Time plane



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A robot trajectory with one single loop

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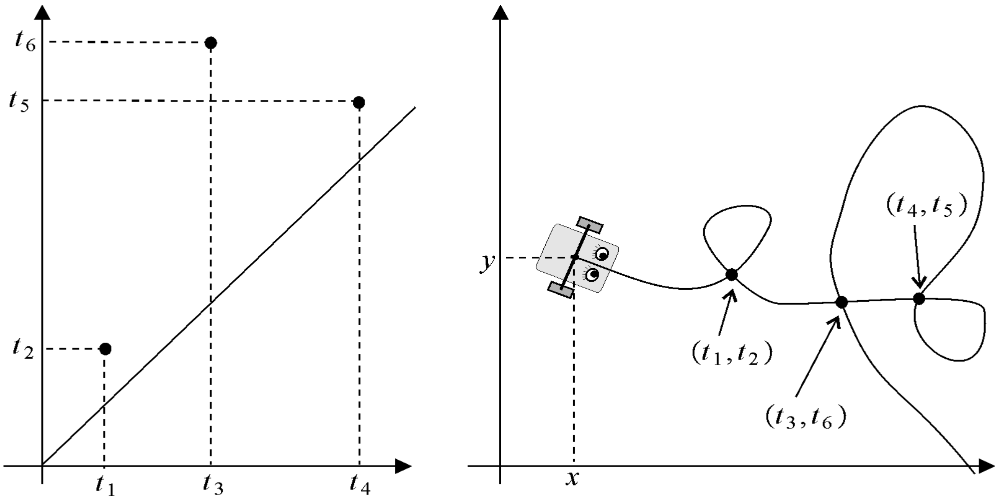


Left: t-plane; Right: trajectory

2 Kernel

Consider a mapping $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$.

$$\ker \mathbf{f} = \{\mathbf{x} \mid \mathbf{f}(\mathbf{x}) = \mathbf{0}\} = \mathbf{f}^{-1}(\mathbf{0}).$$

Example. If a robot has a velocity $\mathbf{v}(t)$. Define

$$\mathbf{f}(\mathbf{t}) = \mathbf{f}(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0}.$$

The loop set is

$$\begin{aligned}\mathbb{T} &= \left\{(t_1, t_2) \in \mathbb{R}^2 \mid \mathbf{f}(t_1, t_2) = \mathbf{0} \text{ and } t_2 > t_1\right\} \\ &= \ker \mathbf{f} \cap \left\{(t_1, t_2) \in \mathbb{R}^2 \mid t_2 > t_1\right\}.\end{aligned}$$

The kernel of an interval function $[\mathbf{f}] : \mathbb{R}^n \longrightarrow \mathbb{IR}^n$ is

$$\mathbb{X} = \ker [\mathbf{f}] = \bigcup_{\mathbf{f} \in [\mathbf{f}]} \ker \mathbf{f} = \{\mathbf{x} \in [\mathbf{x}] \subset \mathbb{R}^n \mid \mathbf{0} \in [\mathbf{f}]\,(\mathbf{x})\}\,.$$

Example 1. With all uncertainties, an automonous robot satisfies

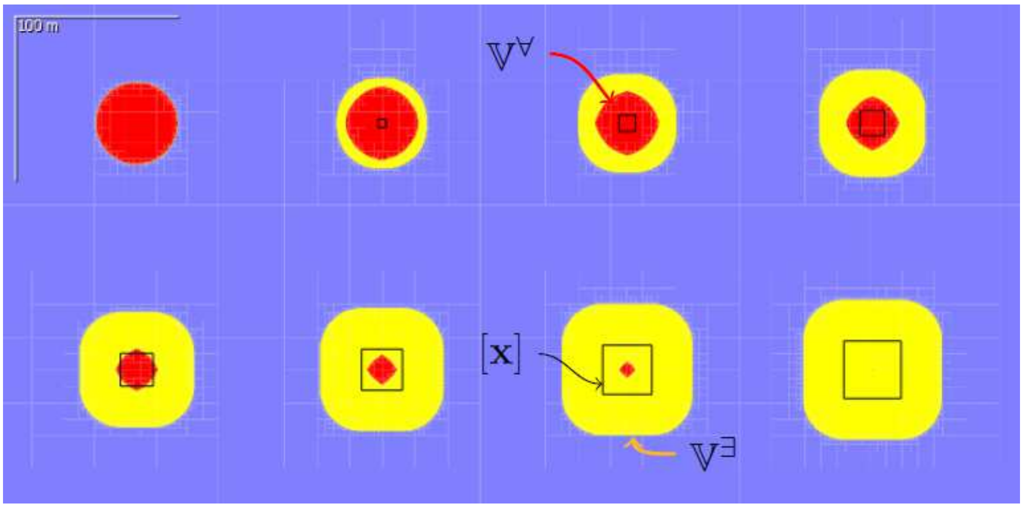
$$\dot{\mathbf{x}} \in [\mathbf{f}] (\mathbf{x}) \, .$$

The set of all feasible equilibrium points is

$$\mathbb{X} = \mathsf{ker} \, [\mathbf{f}] \, .$$

Example 2. Consider the set

$$\begin{aligned}\mathbb{X}^{\exists} &= \{\mathbf{x} \mid \exists \mathbf{p} \in [\mathbf{p}]\,,\ \|\mathbf{x}-\mathbf{p}\| \in [d]\}\,. \\ &= \mathsf{ker}\,[\mathbf{f}]\,.\end{aligned}$$



100 m
$\mathbb{V}^{\forall}$
$[\mathbf{x}]$
$\mathbb{V}^{\exists}$

Problem. Find an inner and an outer approximation of $\mathbb{X} = \ker[\mathbf{f}]$:

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

As a consequence,

$$\ker \mathbf{f}^* \subset \mathbb{X}^+.$$

3 Characterization of the kernel

We have,

$$\begin{aligned}\mathbf{0} &\in [\mathbf{f}](\mathbf{x}) \Leftrightarrow \mathbf{0} \in \left[\mathbf{f}^{-}(\mathbf{x}), \mathbf{f}^{+}(\mathbf{x})\right] \\ &\Leftrightarrow \mathbf{f}^{-}(\mathbf{x}) \leq \mathbf{0} \leq \mathbf{f}^{+}(\mathbf{x}) \;\Leftrightarrow\; \underbrace{\begin{pmatrix} -\mathbf{f}^{-}(\mathbf{x}) \\ \mathbf{f}^{+}(\mathbf{x}) \end{pmatrix}}_{=\boldsymbol{\psi}(\mathbf{x})} \geq \mathbf{0}.\end{aligned}$$

Thus

$$\mathbb{X} = \mathsf{ker}\,[\mathbf{f}] = \boldsymbol{\psi}^{-1}\left(\mathbb{R}^{2n}\right).$$

4 Interval arithmetic

Problem. Given $f : \mathbb{R}^n \longrightarrow \mathbb{R}$, a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that$\forall \mathbf{x} \in [\mathbf{x}]\,, f\left(\mathbf{x}\right) \geq 0$..

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= [1,8], \\ [-1,3]\cdot[2,5] &= [-5,15], \\ \mathsf{abs}\left([-7,1]\right) &= [0,7] \end{aligned}$$

If f is given

Algorithm f(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: y)

$z := x_1$;
for $k := 0$ to 100
    $z := (\cos x_2) \cdot (\sin(z) + kx_3)$;
next;
$y := \sin(zx_1)$;

Its interval extension is

Algorithm $[f]$(in: $[\mathbf{x}] = ([x_1], [x_2], [x_3])$, out: $[y]$)

$[z] := [x_1]$;
for $k := 0$ to 100
    $[z] := (\cos [x_2]) \cdot (\sin ([z]) + k \cdot [x_3])$;
next;
$[y] := \sin([z] \cdot [x_1])$;

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0$$

5 Loop detection

The robot knows a box $[\mathbf{v}](t)$ which contains $\mathbf{v}(t)$ for each. The loop set is

$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\text{max}}]^2 \mid \exists \mathbf{v} \in [\mathbf{v}], \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0} \right\}$$

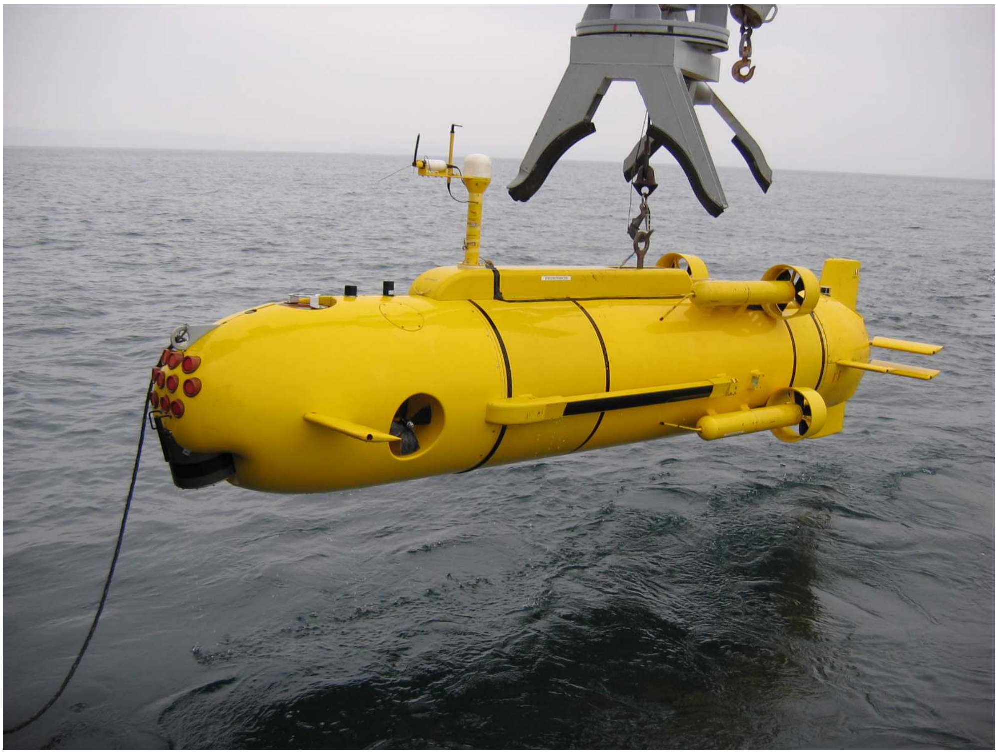
If

$$[\mathbf{f}]\left(\mathbf{t}\right) = \left[\int_{t_1}^{t_2} \mathbf{v}^{-}(\tau)d\tau, \int_{t_1}^{t_2} \mathbf{v}^{+}(\tau)d\tau\right].$$

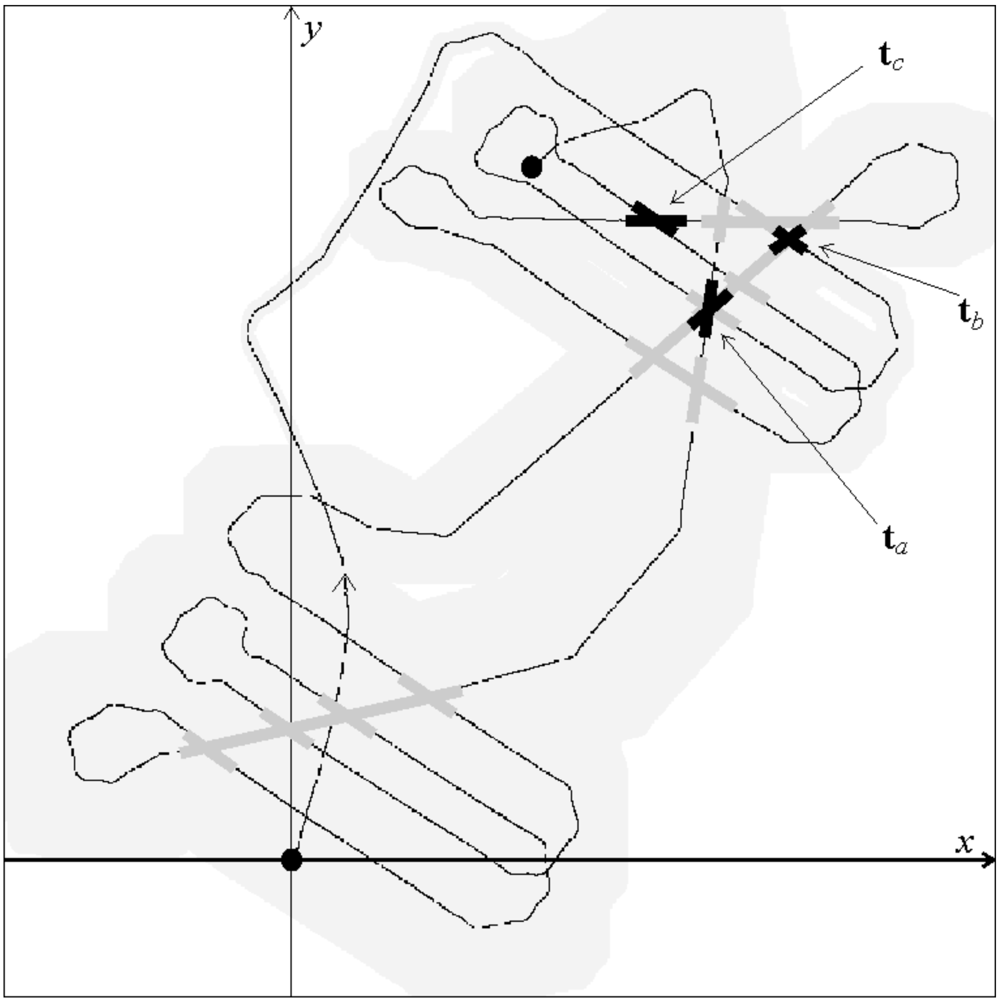
Thus

$$\mathbb{T} = \left\{\mathbf{t} \in [0, t_{\mathsf{max}}]^2, \mathbf{0} \in [\mathbf{f}]\left(\mathbf{t}\right)\right\} = \ker\left[\mathbf{f}\right].$$

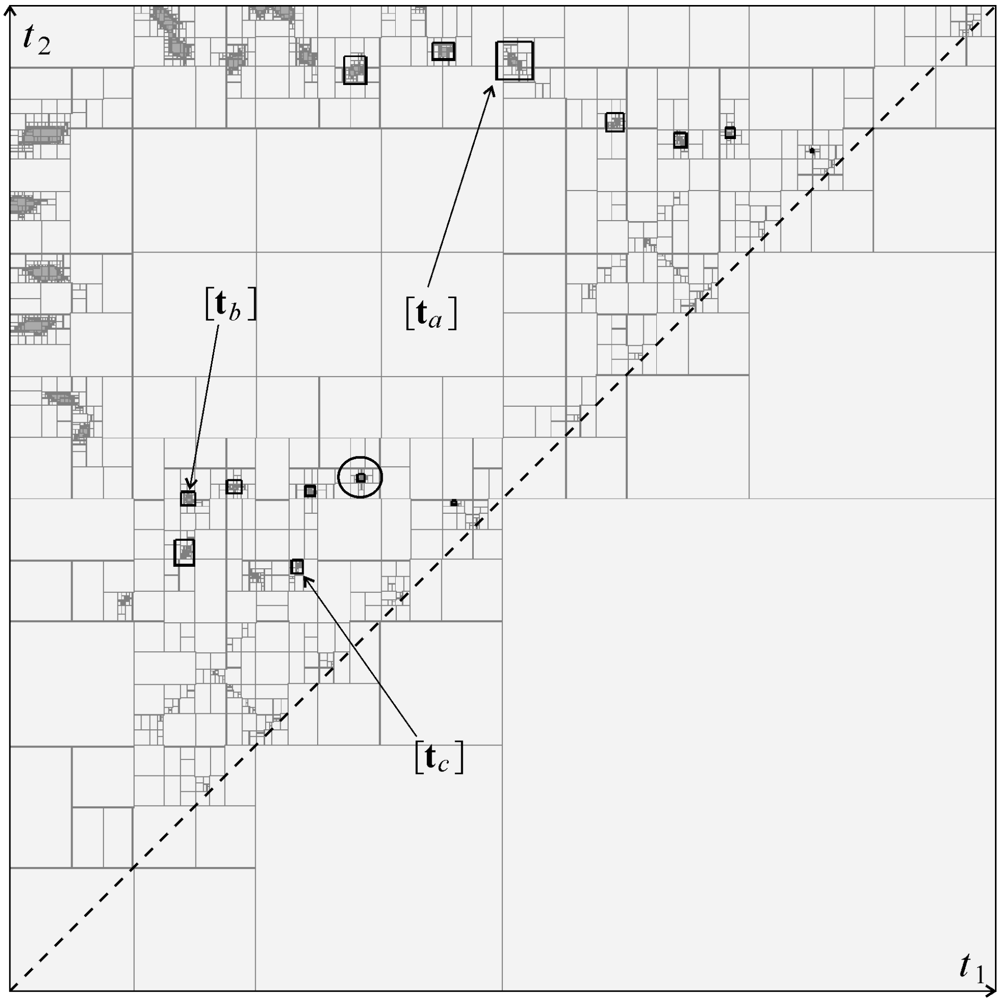
6 Test-case



Redermor, built by GESMA (Groupe d'Etude Sous-Marine de l'Atlantique)



Tube enclosing the trajectory of the robot. The 14 black/gray crosses correspond to detected loops.



Inner and outer approximation of $\mathbb{T}$