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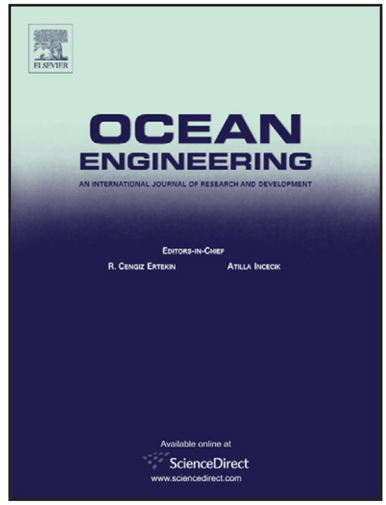

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Nonlinear Path-Following Control of an AUV

Lionel Lapierre*, Didik Soetanto

Abstract

A new type of control law is developed to steer an autonomous underwater vehicle (AUV) along a desired path. The methodology adopted for path-following deals explicitly with vehicle dynamics. Furthermore, it overcomes stringent initial condition constraints that are present in a number of path-following control strategies described in the literature. Controller design builds on Lyapunov theory and backstepping techniques. The resulting nonlinear feedback control law yields convergence of the path-following error trajectory to zero. Simulation results illustrate the performance of the control system proposed.

Index Terms

Nonlinear Control, Path-following, Underactuated Vehicles, Autonomous Underwater Vehicles.

I. INTRODUCTION

RAPID progress in marine robotics is steadily affording scientists advanced tools for ocean exploration and exploitation. However, much work remains to be done before marine robots can roam the oceans freely, acquiring scientific data on the temporal and spatial scales that are naturally imposed by the phenomena under study. To meet these goals, robots must be equipped with systems to steer them accurately and reliably in the harsh marine environment. For this reason, there has been considerable interest over the last few years in the development of advanced methods for marine vehicle motion control. Namely, point stabilization, trajectory tracking, and path-following control.

Point stabilization refers to the problem of steering a vehicle to a final target point, with a desired orientation. Trajectory tracking requires a vehicle to track a time-parameterized reference curve. Finally, path-following control aims at forcing a vehicle to converge to and follow a desired spatial path, without any temporal specifications. The latter objective occurs for example when it is required that an AUV examine an area by performing a ”lawn mowing” maneuver along desired tracks with great accuracy, at speeds determined by a scientific end-user. The underlying assumption in path-following control is that the vehicle's forward speed conforms to a desired speed profile, while the controller acts on the vehicle's orientation to steer it to the path. Typically, smoother convergence to a path is achieved when path-following strategies are used instead of trajectory tracking control laws, and the control signals are less likely to reach saturation.

This paper proposes a new methodology for the design of path following systems for AUVs. The reader is referred to the work of Micaelli and Samson (Micaelli and Samson, 1992 and 1993) and the references therein for related ground-breaking work in the field of land robots, where powerful nonlinear path-following control structures were introduced. It is important to remark that even though the problem of path-following is essentially solved for land vehicles, the same does not hold true for marine craft. This is due to the fact that dynamics play a key role in the motion of the latter, thus requiring the development of methodologies for accurate path-following that take explicitly into account the presence of possibly complex, nonlinear hydrodynamic terms. This is in striking contrast with land vehicles, where methodologies that build on pure vehicle kinematics are often adequate for control.

The present paper builds on previous results obtained by Encarnação and Pascoal (Encarnação and Pascoal, 2000) and in (Encarnação *et al.*, 2000), where the results in (Micaelli and Samson, 1992 and 1993) were extended to deal with the control of marine vehicles in three-dimensional space, and to address explicitly the presence of non-negligible marine vehicle dynamics. The methodology for path-following proposed in (Encarnação *et al.*, 2000) can be easily understood by recalling that the total velocity vector of an AUV is not necessarily aligned with the vehicle's main axis, as is the case for wheeled robots (AUVs sideslip). However, by drawing a simple analogy between the problems of path-following for wheeled robots and AUVs, the latter can be cast as the equivalent problem of aligning the total AUV velocity vector with the tangent to the path by manipulating the vehicle's yaw rate. It is important to remark that in spite of its broader scope of application, the results in (Encarnação *et al.*, 2000), inherit the major shortcoming already present in the path following control strategy for wheeled robots described for example in (Micaelli and Samson, 1993) : *the initial position error must be smaller than the smallest radius of curvature present in the path*.

The work reported in this paper lifts this restriction entirely. This is achieved by controlling explicitly the progression rate of a 'virtual target' to be tracked along the path, thus bypassing the problems that arise when the position of the virtual target is simply defined by the projection of the actual vehicle onto that path. See the work of Soetanto *et al.*, (Soetanto *et al.*, 2003) where a similar technique was first proposed for wheeled robots. This design procedure effectively creates an extra degree of freedom, that can then be explored to avoid the *singularities* that occur when the distance to path is not well defined (this

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occurs for example when the vehicle is located exactly at the center of curvature of a circular path). Controller design starts at a kinematic level and evolves to a dynamic setting using backstepping techniques. The resulting control strategy yields global convergence of the actual path of the vehicle to the desired path.

As remarked in (Soetanto *et al.*, 2003), the idea of exploring the extra degree of freedom that comes from controlling the motion of a virtual target along a path seems to have appeared for the first time in (Casalino *et al.*, 1995), for the control of wheeled robots. This idea was later extended to the control of marine craft in (Aicardi *et al.*, 2001). However, none of these references addresses the issues of vehicle dynamics. Furthermore, the methodologies adopted for control system design in (Casalino *et al.*, 1995) and (Aicardi *et al.*, 2001) build on an entirely different technique, that requires the introduction of a nonsingular transformation in the original error space. Interestingly enough, a very recent publication explores the same concept of a virtual target for path following in wheeled robots, (Diaz *et al.*, 2002).

The paper is organized as follows. Section II formulates the problem of path-following control for an AUV. Section III develops a nonlinear path-following control law that deals explicitly with vehicle dynamics. The performance of the control system proposed is illustrated in simulation in Section IV. Finally, Section V contains the conclusions, and describes some problems that warrant further research.

II. PROBLEM FORMULATION

This section introduces the simplified dynamic model of an AUV in the horizontal plane, and provides a rigorous formulation of the problem of steering it along a desired path.

The type of AUV considered in this paper is equipped with two identical back thrusters, mounted symetrically with respect to its longitudinal axis. Thus, the vehicle is underactuated since it lacks a lateral thruster. The common and differential modes of the thrusters generate a force F along the vehicle's longitudinal axis and a torque Γ about its vertical axis, respectively. In this study, a full dynamic model of the INFANTE AUV operated by the Institute for Systems and Robotics (ISR) of Lisbon will be used, see Figure 2 and Table II. It is assumed that only the back thrusters are used to maneuver in the horizontal plane.

A. Vehicle Modeling. Kinematics and Dynamics.

The following notation will be used in the sequel. The symbol $\{A\} := \{\mathbf{x_A}, \mathbf{y_A}, \mathbf{z_A}\}$ denotes a reference frame with origin O_A and unit vectors $\mathbf{x_A}, \mathbf{y_A}$, and $\mathbf{z_A}$. Given two reference frames $\{A\}$ and $\{B\}$, ${}^B_AR$ is the rotation matrix from $\{B\}$ to $\{A\}$. Following standard practice, the general kinematic and dynamics equations of a vehicle can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$, as depicted in Figure 1. Let Q denote the center of mass of the vehicle, which we assume is coincident with O_B, and let $\mathbf{q} = [x, y, 0]^T$ be the position of Q in $\{U\}$. Further let ψ_B denote the yaw angle that parametrizes the rotation matrix from $\{B\}$ to $\{U\}$. Let $\mathbf{v_t} = [u, v, 0]^T$ be the velocity of Q in $\{U\}$ expressed in $\{B\}$, where u and v are the longitudinal (surge) and transverse (sway) velocities, respectively.

With this notation, the kinematic equations of the AUV can be written as

$$\begin{aligned} \dot{x} &= u\cos(\psi_B) - v\sin(\psi_B) \\ \dot{y} &= u\sin(\psi_B) + v\cos(\psi_B) \\ \dot{\psi}_B &= r \end{aligned} \tag{1}$$

where r is the vehicle's angular speed (yaw rate). Assuming u is never equal to zero, define the side-slip angle $\beta = \arctan(v/u)$ and consider the reference frame $\{W\}$ that is obtained from $\{B\}$ by rotating it around the $\mathbf{z_B}$ axis through angle β in the positive direction . The above equations can then be re-written to yield

$$\begin{aligned} \dot{x} &= v_t\cos(\psi_W) \\ \dot{y} &= v_t\sin(\psi_W) \\ \dot{\psi}_W &= r + \dot{\beta} \end{aligned} \tag{2}$$

where $\psi_W = \psi_B + \beta$ and v_t is the x_W component of the total vehicle velocity expressed in $\{W\}$. Clearly, $v_t = ||\mathbf{v_t}|| = (u^2 + v^2)^{1/2}$. In the aircraft literature $\{W\}$ is called the wind frame and will henceforth be called the flow frame. Notice how the choice of a new frame simplified the first two kinematic equations, and brought out their similarities with those of a wheeled robot. See (Micaelli and Samson 1993), and (Soetanto *et al.*, 2003).

Neglecting the equations in heave, roll and pitch, the simplified equations for surge, sway and yaw can be written as in (Fossen, 1994) and (Silvestre, 2000).

$$\begin{aligned} F &= m_u\dot{u} + d_u \\ 0 &= m_v\dot{v} + m_{ur}ur + d_v \\ \Gamma &= m_r\dot{r} + d_r \end{aligned} \tag{3}$$

where

$$\begin{array}{lllllll} m_u & = & m - X_{\dot{u}} & d_u & = & -X_{uu}u^2 - X_{vv}v^2 \\ m_v & = & m - Y_{\dot{v}} & d_v & = & -Y_v uv - Y_{v|v|}v|v| \\ m_r & = & I_z - N_{\dot{r}} & d_r & = & -N_v uv - N_{v|v|}v|v| \\ m_{ur} & = & m - Y_r & & & -N_r ur. \end{array}$$

The symbols m and I_z denote the mass and moment of inertia of the AUV respectively, $X_{\{.\}}, Y_{\{.\}}$, and $N_{\{.\}}$ are classical hydrodynamic derivatives, and $[F \quad \Gamma]^T$ defines the input vector of force and torque that is applied to the AUV. The model presented in this paper is based on the model of the INFANTE AUV described in (Silvestre, 2000), to which the reader is referred for complete details.

B. Path-following. Error Coordinates.

The solution to the problem of path-following proposed here builds on the following intuitive explanation, see Figure 1): a simple path-following controller should compute i) the distance between the vehicle's center of mass Q and the closest point P on the path, and ii) the angle between the vehicle's total velocity vector $\mathbf{v_t}$ and the tangent to the path at P, and reduce both to zero. This motivates the development of the 'kinematic' model of the vehicle in terms of a Serret-Frenet frame $\{$F$\}$ that moves along the path; $\{$F$\}$ plays the role of the body axis of a 'virtual target vehicle' that should be tracked by the 'real vehicle'. Using this set-up, the abovementioned distance and angle become the coordinates of the error space where the control problem is formulated and solved. In this paper, however, a Frenet frame $\{$F$\}$ that moves along the path to be followed is used with a significant difference: *the Frenet frame is not attached to the point on the path that is closest to the vehicle*. Instead, the origin $O_F = P$ of $\{$F$\}$ along the path is made to evolve according to a conveniently defined control law, effectively yielding an extra controller design parameter. As will be seen, this seemingly simple procedure is instrumental in lifting the stringent initial condition constraints that are patent in (Micaelli and Samson, 1993), for path-following in wheeled robots and in (Encarnação *et al.*, 2000), for marine vehicles. The notation that follows is by now standard.

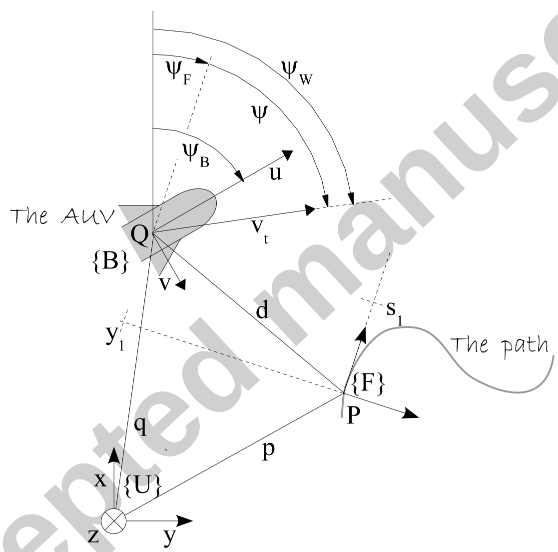

Fig. 1. Path-following: reference frames

Consider Figure 1, where P is an arbitrary point on the path to be followed. Associated with P, consider the corresponding Serret-Frenet frame $\{$F$\}$. The signed curvilinear abscissa of P along the path is denoted s. Clearly, Q can either be expressed as $\mathbf{q} = [x, y, 0]^T$ in $\{$U$\}$ or as $[s_1, y_1, 0]^T$ in $\{$F$\}$. Stated equivalently, Q can be given in $(x, y, 0)$ or $(s_1, y_1, 0)$ coordinates. Let

$$R = \begin{bmatrix} \cos\psi_F & \sin\psi_F & 0 \\ -\sin\psi_F & \cos\psi_F & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be the rotation matrix from $\{$U$\}$ to $\{$F$\}$, parameterized locally by the angle ψ_F. Define $\omega_F = \dot{\psi}_F$. Then,

$$\begin{cases} \omega_F = \dot{\psi}_F = c_c(s)\dot{s} \\ \dot{c}_c(s) = g_c(s)\dot{s} \end{cases} \tag{4}$$

where $c_c(s)$ and $g_c(s) = \frac{dc_c(s)}{ds}$ denote the path curvature and its derivative, respectively. The velocity of P in $\{$U$\}$ can be expressed in $\{$F$\}$ to yield

$$\left(\frac{d\mathbf{p}}{dt}\right)_F = \begin{bmatrix} \dot{s} \\ 0 \\ 0 \end{bmatrix}.$$

It is also straightforward to compute the velocity of Q in {U} as

$$\left(\frac{d\mathbf{q}}{dt}\right)_U = \left(\frac{d\mathbf{p}}{dt}\right)_U + R^{-1}\left(\frac{d\mathbf{d}}{dt}\right)_F + R^{-1}(\omega_F \times \mathbf{d})$$

where $\mathbf{d}$ is the vector from P to Q. Multiplying the above equation on the left by R gives the velocity of Q in {U} expressed in {F} as

$$R\left(\frac{d\mathbf{q}}{dt}\right)_U = \left(\frac{d\mathbf{p}}{dt}\right)_F + \left(\frac{d\mathbf{d}}{dt}\right)_F + \omega_F \times \mathbf{d}. \tag{5}$$

Using the relations

$$\left(\frac{d\mathbf{q}}{dt}\right)_U = \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix},$$

$$\left(\frac{d\mathbf{d}}{dt}\right)_F = \begin{bmatrix} \dot{s}_1 \\ \dot{y}_1 \\ 0 \end{bmatrix},$$

and

$$\begin{aligned} \omega_F \times \mathbf{d} &= \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_F = c_c(s)\dot{s} \end{bmatrix} \times \begin{bmatrix} s_1 \\ y_1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -c_c(s)\dot{s}y_1 \\ c_c(s)\dot{s}s_1 \\ 0 \end{bmatrix} \end{aligned}$$

equation (5) can be rewritten as

$$R\begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{s}(1 - c_c(s)y_1) + \dot{s}_1 \\ \dot{y}_1 + c_c(s)\dot{s}s_1 \\ 0 \end{bmatrix}.$$

Solving for $\dot{s}_1$ and $\dot{y}_1$ yields

$$\begin{cases} \dot{s}_1 = \begin{bmatrix} \cos\psi_F & \sin\psi_F \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \dot{s}\,(1 - c_c y_1) \\ \dot{y}_1 = \begin{bmatrix} -\sin\psi_F & \cos\psi_F \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - c_c\dot{s}s_1. \end{cases} \tag{6}$$

Finally, replacing the top two equations of (2) in (6) and introducing the variable $\psi = \psi_W - \psi_F$ gives the 'kinematic' model of the AUV in (s, y) coordinates as

$$\begin{cases} \dot{s}_1 = -\dot{s}\,(1 - c_c y_1) + v_t \cos\psi \\ \dot{y}_1 = -c_c\dot{s}s_1 + v_t \sin\psi \\ \dot{\psi} = \omega_W - c_c\dot{s} \end{cases} \tag{7}$$

where $\omega_W = \dot{\psi}_W = r + \dot{\beta}$.

At this point it is important to notice that in (Micaelli and Samson, 1993) and (Encarnação *et al.*, 2000), the point P is defined by the projection of Q onto the path, assuming the projection is well defined. In other terms, the kinematic model considered in (Micaelli and Samson, 1993) and (Encarnação *et al.*, 2000) is equivalent to the equations (7) with $s_1 = 0$. One is then forced to solve for $\dot{s}$ in the equation above when s_1 is forced to 0. However, by doing so $1 - c_c y_1$ appears in the denominator, thus creating a singularity at $y_1 = \frac{1}{c_c}$. As a result, the control laws derived in (Micaelli and Samson, 1993) and (Encarnação *et al.*, 2000), require that the initial position of Q be restricted to a tube around the path, the radius of which must be less than $\frac{1}{c_{c,max}}$, where $c_{c,max}$ denotes the maximum curvature of the path. Clearly, this constraint is very conservative since the occurrence of a large $c_{c,max}$ in just a small section of the path will impose a rather strict constraint on the vehicle's initial position, even if it happens to start in a region that is far away from the 'problematic' section. By making s_1 not necessarily equal to zero, a virtual target that is not coincident with the projection of the vehicle onto the path is created, thus introducing an extra degree of freedom for controller design. By specifying how fast the newly-defined target moves, the occurrence of a singularity at $y_1 = \frac{1}{c_c}$ is removed.

C. Problem Formulation

With the above notation, the problem under study can be formulated as follows:

Consider the AUV model with kinematic and dynamic equations given by (1) and (3), respectively. Given a path to be followed and a desired profile $u_d > u_{min} > 0$ *for the surge speed* u*, derive feedback control laws for the force* F*, torque* Γ*, and rate of evolution* $\dot{s}$ *of the curvilinear abscissa* s *of the 'virtual target' point* P *along the path so that* y_1*,* s_1*,* ψ*, and* $u - u_d$ *tend to zero asymptotically*.

III. NONLINEAR PATH-FOLLOWING CONTROLLER DESIGN

This section introduces a nonlinear closed-loop control law to steer the dynamic model of an AUV described by (1)-(3) along a desired path. Controller design builds on previous work in (Micaelli and Samson, 1993) and (Encarnação *et al.*, 2000), on path-following control, and relies heavily on backstepping techniques. The reader will find in (Krstić *et al.*, 1995) a lucid exposition of interesting theoretical and practical issues involved in backstepping. Controller design is done in two steps. The first step yields a kinematic controller by adopting the yaw rate $r = \dot{\psi}_B$ as a 'virtual' control input, and by assuming that the actual surge speed equals the desired speed u_d. The second step addresses the vehicle dynamics, builds on the kinematic controller derived, and uses backstepping techniques to obtain control laws for the input variables F and Γ.

A. Controller Design using the Kinematic Model

This section derives a kinematic controller for the AUV. As in (Micaelli and Samson, 1993), we let

$$\delta(y_1) = -\psi_a \frac{e^{2k_\delta y_1} - 1}{e^{2k_\delta y_1} + 1} \tag{8}$$

be a desired approach angle parameterized by $k_\delta > 0$ and $0 < \psi_a < \pi/2$, satisfying $y_1\delta(y_1) \leq 0$ for all y_1. The approach angle is instrumental in shaping transient maneuvers during the path approach phase.

Remark : as mentioned in (Micaelli and Samson, 1993) for wheeled vehicles, the approach angle is generally chosen as $\delta'(y_1, u) = -sign(u)\delta(y_1)$, where u is the forward velocity of the wheeled robot. Note that the nonholonomic constraint makes the side-slip velocity null, so $v_t = u$ in this situation. This choice does not guarantee the differentiability of δ' around $u = 0$. In the case of an AUV, the condition $u(t) > 0, \forall t$, is necessary for controllability reasons. Effectively this kind of vehicle is generally actuated via control surfaces that generate an action on the body if there is non-null relative fluid-flow velocity. This justifies the choice of the approach angle expression, and the necessary assumptions that will be used in the proof.

Proposition 1: *Consider the kinematic model of an AUV described in (1) and the corresponding path-following error model (7). Let the approach angle* $\delta(y_1)$ *be defined as in (8). Assume that the surge velocity of the vehicle is such that* $u = u_d > 0$*. Suppose that the path to be followed is parametrized by its curvilinear abscissa* s*, and assume that for each* s *the variables* $\psi,\ s_1,\ y_1, and\, c_c$ *are well defined. Then the kinematic control law*

$$U_{kin} = \begin{cases} r = \dot{\delta} - \dot{\beta} - k_1(\psi - \delta) + c_c(s)\dot{s} \\ \dot{s} = \cos\psi v_t + k_2 s_1 \end{cases} \tag{9}$$

(where k_1 *and* k_2 *are arbitrary positive constants) drives* y_1*,* s_1*, and* ψ *asymptotically to zero.*

At this point, one should notice that the equation (9) appears in a non-causal form. Indeed, the term $\dot{\beta}$ is hiding acceleration terms, and through them a dependance on r itself. This is due to the coupling term m_{ur} appearing in the side-slip dynamics ($\dot{v}$).

Nevertheless, the consideration of the dynamic model (3) yields an algebraic solution for the expression of the kinematic control r as described below:

$$r = \frac{1}{1 - \frac{m_{ur}}{m_v}(\cos\beta)^2} \left(\dot{\delta} + \frac{1}{v_t^2}(\dot{u}v + \frac{ud_v}{v_t^2}) - k_1(\psi - \delta) + c_c(s)\dot{s} \right) \tag{10}$$

Note that the above control is causal and well defined if

$$\frac{m_{ur}}{m_v} = \frac{m - Y_r}{m - Y_{\dot{v}}} < 1. \tag{11}$$

Close examination of the hydrodynamic parameters of an AUV as in (Lewis, 1989) reveals that

- $Y_{\dot{v}}$ is always negative.
- Y_r is positive if stern dominates.
- Y_r is negative if bow dominates.

Thus, in the case of a stern-dominant vehicle such as the one considered in this study, the control computation is well posed. The implementation of this control requires the consideration of the equation (10), where a surge acceleration estimation $\dot{u}$ is necessary. Nevertheless, at this kinematic level, one could restrict the domain of application to the case where the system is traveling with a constant forward velocity $u = u_d > 0$. This makes the expression (10) simpler since the acceleration terms disappear.

The following proof will extensively use Barbalat's lemma and the LaSalle's invariance principle stated as follows.

Barbalat's lemma : *if $f(t)$ is a double differentiable function such that $f(t)$ is finite as t goes to ∞, and such that $\dot{f}(t)$ is uniformly continuous, then $\dot{f}(t)$ tends to 0 as t tends to ∞*

Uniform continuity sufficient condition : *$\dot{f}(t)$ is uniformly continuous if $\ddot{f}(t)$ exists and is bounded.*

LaSalle's Invariance Principle : *let Ω be a positively invariant set of the system described in (1) and (3). Suppose that every solution starting in Ω converges to a set $E \subset \Omega$ and let M be the largest invariant set contained in E. Then every bounded solution starting in Ω converges to M as t tends to ∞.*

For details of Barbalat's lemma and its application, please refer to (Slotine and Li, 1995). For demonstration and application of LaSalle's theorem, please refer to (Sepulchre *et al.*, 1997) and (Khalil, 2002). Note that the application of LaSalle's theorem is restricted to autonomous systems. In our situation, the fact that the desired forward velocity is a constant allows our system to be considered as autonomous.

Proof: The following proof is made under the assumption that the system is traveling with a constant surge velocity $u = u_d > 0$. This allows us to consider the system as autonomous, and in turn allows for the application of LaSalle's invariance principle. The proof is structured in three parts. First, we show that the system asymptotically follows the reference approach angle δ. Then we show that this reference asymptotically drives the AUV onto the path. And finally, we use LaSalle's invariance principle to concatenate the two previous convergence properties.

- Consider the candidate Lyapunov function $V_1 = \frac{1}{2}(\psi - \delta)^2$. The control law

$$r = \dot{\delta} - \dot{\beta} - k_1(\psi - \delta) + c_c(s)\dot{s}$$

  makes $\dot{V}_1 = -k_1(\psi - \delta)^2 \leq 0$. Since V_1 is a positive and monotonically decreasing function, $\lim_{t\to\infty} V_1(t)$ exists and is finite. Moreover, since $\ddot{V}_1 = -2k_1\dot{V}_1$, $\ddot{V}_1$ is bounded and therefore $\dot{V}_1$ is uniformly continuous. Then an application of Barbalat's lemma allows for the conclusion that $\lim_{t\to\infty} \dot{V}_1 = 0$. The system is asymptotically converging to a compact set E defined by $\dot{V}_1 = 0$. Therefore, the related variables y_1, s_1, s and ψ are bounded.
- Examine now the motion of the feedback control system restricted to E. To do this, consider the candidate Lyapunov function candidate $V_E = \frac{1}{2}(s_1^2 + y_1^2)$ and compute its derivative $\dot{V}_E = y_1 v_t \sin\delta - k_2 s_1^2 \leq 0$, according to the definition of δ in (8). Considering the previous expression of $\dot{V}_E$ and the equations (7), $\ddot{V}_E$ is bounded. Then Barbalat's lemma allows for the conclusion that $\lim_{t\to\infty} \dot{V}_E = 0$. This in turn implies that all trajectories in E satisfy $\lim_{t\to\infty} y_1 = 0$ and $\lim_{t\to\infty} s_1 = 0$.
- We now apply LaSalle's invariance principle. Let $\Omega = \Re^2$. The first part of the proof showed that every solution starting in Ω asymptotically converges to E. The second step showed that the largest invariant set of E is $M = [(s_1, y_1) = 0^2]$. Then every bounded solution starting in Ω converges to 0 as t tends to ∞, or in other terms,

$$\begin{cases} \lim_{t\to\infty} s_1 & = & 0 \\ \lim_{t\to\infty} y_1 & = & 0 \\ \lim_{t\to\infty} \psi & = & 0. \end{cases} \tag{12}$$

End of proof.

B. The Dynamic controller

The above feedback control law applies to the kinematic model of the AUV only. However, using backstepping techniques, this control law can be extended to deal with the vehicle dynamics. In the kinematic design the total velocity $v_t(t)$ of the vehicle was left free, but implicitly dependent on a desired profile u_d for surge speed $u(t)$. In the dynamic design the variable u will be explicitly brought into the picture and a control law will be derived so that $u(t) - u_d$ tends to zero. Notice also that the robot's angular speed r was assumed to be a control input. This assumption is lifted by taking into account the vehicle dynamics. The following result holds.

Proposition 2: *Consider the kinematic and dynamic models of an AUV described in (1) and (3) respectively, and the corresponding path-following error model in (7). Let the approach angle $\delta(y_1)$ be defined as in (8) and let a desired speed profile $u_d > u_{min} > 0$ for $u(t)$ be given. Suppose the path to be followed is parametrized by its curvilinear abscissa s, and assume that for each s the variables $\psi,\ s_1,\ y_1, and\, c_c\, \frac{\partial c_c}{\partial s}$ are well defined. Then the dynamic control law*

$$U_{dyn} = \begin{cases} \Gamma & = & m_r\alpha_r - d_r \\ F & = & m_u(\dot{u_d} - k_4(u - u_d)) - d_u \\ \dot{s} = \cos\psi v_t + k_2 s_1 & & \end{cases} \tag{13}$$

where

$$\alpha_r = \ddot{\delta} - \ddot{\beta} - (k_1 + k_3)(\dot{\psi} - \dot{\delta})$$
$$-(k_5 + k_1k_3)(\psi - \delta) + c_c\ddot{s} + \frac{\partial c_c}{\partial s}\dot{s},$$

k_1 through k_5 are arbitrary positive gains, and d_r and d_u are sums of hydrodynamic coefficients, drives y_1, s_1, $u - u_d$, and ψ asymptotically to zero.

The control computation requires further algebraic derivation, as explained in the previous section of the kinematic controller, where the use of the dynamic model is required. The equations (13) contain an evaluation of the side-slip angle acceleration $\ddot{\beta}$. This variable cannot be measured directly, and one must thus resort for its computation to the original dynamic model of the AUV. It is easy to see that

$$\ddot{\beta} = \frac{1}{v_t^2}(\ddot{v}u - \ddot{u}v) - 2\frac{\dot{v}_t}{v_t}\dot{\beta} \tag{14}$$

and that the dynamic model of the AUV can be differentiated to obtain

$$\left\{ \begin{array}{lcl} \ddot{u} & = & \frac{1}{m_u}(\dot{F} - \dot{d}_u) \\ \ddot{v} & = & \frac{1}{m_v}(-m_{ur}\dot{u}r - m_{ur}u\dot{r} - \dot{d}_v) \end{array} \right. . \tag{15}$$

Replacing these equations in the control law and simplifying yields

$$\Gamma = m_r\alpha + d_r$$

where

$$\alpha = \frac{f_\alpha - k_3(r - r_d) - k_5(\psi - \delta)}{1 - \frac{m_{ur}}{m_v}(\cos\beta)^2}$$

and

$$\begin{array}{rcl} f_\alpha & = & \ddot{\delta} - k_1(\dot{\psi} - \dot{\delta}) + c_c\ddot{s} + g_c\dot{s} \\ & & +\frac{\ddot{u}v}{v_t^2} + \frac{2\dot{v}_t\dot{\beta}}{v_t} + \frac{u}{v_t^2}(\frac{m_{ur}}{m_v}\dot{u}r + \frac{\dot{d}_v}{m_v}). \end{array}$$

This equation can be further expanded into simpler terms using the vehicle model. Notice that we recover the previous condition expressed in the equation (11) for the kinematic case.

Proof: Define the virtual control law for r (desired behaviour of r in (9)) as

$$\zeta = \dot{\delta} - \dot{\beta} - k_1(\psi - \delta) + c_c(s)\dot{s}$$

and let $\epsilon = r - \zeta$ be the difference between actual and desired values of r. Set $r = \epsilon + \zeta$ and consider the total candidate Lyapunov function

$$V_2 = k_5V_1 + \frac{1}{2}\epsilon^2 + \frac{1}{2}(u - u_d)^2 \tag{16}$$

with k_5 positive. Tedious but straightforward computation shows that with the control law proposed

$$\begin{array}{rcl} \dot{V}_2 & = & -k_1k_5(\psi - \delta)^2 - k_3\epsilon^2 \\ & & -k_4(u - u_d)^2 \leq 0 \end{array}$$

and that $\ddot{V}_2$ is bounded. The necessary conditions for the application of Barbalat's lemma are met, and we conclude that $lim_{t\to\infty}\dot{V}_2 = 0$. The consequence is that the system is asymptotically converging to a set E' defined by $\dot{V}_2 = 0$, or in other terms $E' := ((\psi - \delta, r - \zeta, u - u_d) \in \Re^3 | (\psi - \delta = 0, r - \zeta = 0, u - u_d = 0))$. As in the previous section, we now study the system trajectories restricted to the set E'. This study has been carried out for the kinematic case, and the previous results may be used to conclude that onto the set E', the system properties imply that $lim_{t\to\infty}(s_1, y_1) = 0^2$. Since our system is autonomous, we use Lasalle's invariance principle to conclude that

$$\left\{ \begin{array}{rcl} \lim_{t\to\infty} u - u_d & = & 0 \\ \lim_{t\to\infty} s_1 & = & 0 \\ \lim_{t\to\infty} y_1 & = & 0 \\ \lim_{t\to\infty} \psi & = & 0 \end{array} \right. . \tag{17}$$

End of proof.

IV. SIMULATION RESULTS

This section illustrates the performance in simulation of the derived path-following control law. Table II summarizes the key parameters of the AUV model used. This is a simple modification of the Infante AUV model described in (Silvestre, 2000), to account for the fact that only thrusters are used to maneuver in the horizontal plane.

The path is designed in Cartesian space (cf. table I) and we assume a parameterization that allows the computation (given s) of:

- $\psi_F(s)$: the global heading of the virtual target,
- $c_c(s)$: the path curvature at the target position,
- $\frac{\partial c_c(s)}{\partial s}$: the curvilinear derivative of the curvature at the target position,

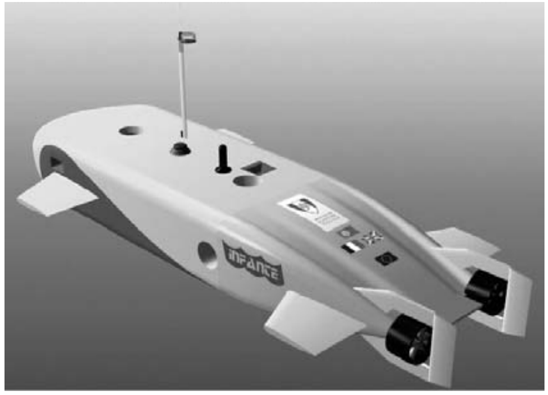

Fig. 2. Infante

- $x_s(s)$ and $y_s(s)$, the absolute location of the virtual target.

  We have chosen a polynomial parameterization of the form

$$x_s(\mu) = \sum_{i=0}^{n} a_i \mu^i \; ; \; y_s(\mu) = \sum_{i=0}^{n} b_i \mu^i .$$

Assuming we have a precise estimation of the function $\mu(s)$, and given s, we compute

$$\begin{array}{ccc} \psi_F(s) & = & \arctan \frac{(y_s)'}{(x_s)'} \\ c_c(s) & = & \frac{\partial \psi_F(s)}{\partial \mu} \frac{d\mu}{ds} \\ \frac{\partial c_c(s)}{\partial s} & = & \frac{\partial c_c(s)}{\partial \mu} \frac{d\mu}{ds} \\ x_s(\mu(s)) & ; & y_s(\mu(s)) \\ (x_s)' = \frac{dx_s}{d\mu} & ; & (y_s)' = \frac{dy_s}{d\mu} . \end{array}$$

The estimation of the function $\mu(s)$ is achievd by integration of

$$\frac{d\mu}{ds} = \frac{1}{\sqrt{[(x_s)']^2 + [(y_s)']^2}} .$$

$a_0 = 0$	$a_1 = 0.866$	$a_2 = -0.02$	$a_3 = 10^{-5}$	$a_4 = 1.5.10^{-6}$
$b_0 = 0$	$b_1 = 0.5$	$b_2 = -5.10^{-4}$	$b_3 = 10^{-5}$	$b_4 = 10^{-7}$

TABLE I

THE PATH PARAMETERS

The reference and actual robot paths are shown in Figure 3. The desired surge speed u_d was set to $1ms^{-1}$. The controller design parameters are displayed in Table III.

Figures 3, 4, 5 and 6 shows the results of the simulation. Notice in Figure 4 how the coordinates s_1 and y_1 tend to zero asymptotically. This is equivalent to stating that: i) the position of the virtual target (origin of frame $\{F\}$(s) along the path) approaches the projection of the AUV on that path, and ii) the lateral distance of the AUV to the path is driven to zero. Notice also in Figure 5 that the actual surge velocity $u(t)$ converges to $u_d = 1ms^{-1}$.

V. DISCUSSION

The study that resulted in the development previously described , opened some interesting issues, that should be explicitly addressed. This section proposes a brief discussion of these questions.

A. The Virtual Target

The idea of the virtual target was implicit in some previous publications (Aicardi *et al.*, 2001, Skjene *et al.*, 2002, Diaz *et al.*, 2002), where the virtual target control law was 'intuitively chosen' before establishing global robot control. The originality of this present work is to show that the virtual target control law can be extracted during robot control design, guaranteeing a non-singular description of the problem and of system performance. Using this design process, it can be imagined that virtual target control law can permit the addressing of other problematic properties, and in turn solves other kinds of problem.

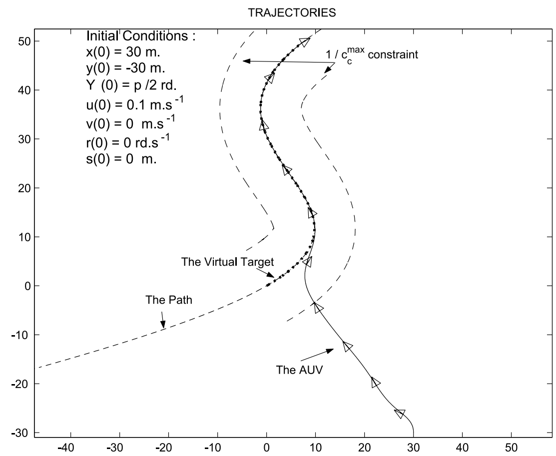

Fig. 3. Robot trajectory

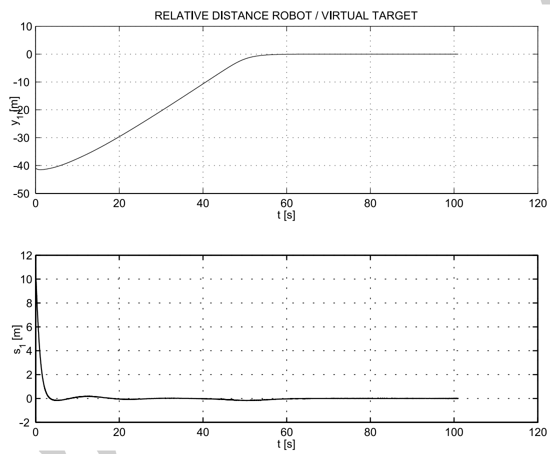

Fig. 4. Relative distance Robot/Virtual Target

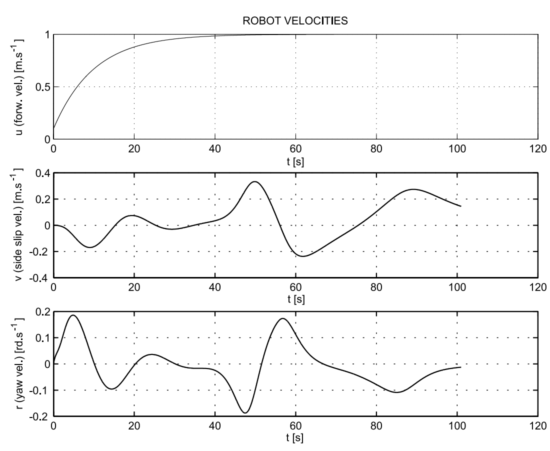

Fig. 5. Robot velocities

$m = 2234.5Kg$	$I_z = 2000N.m^2$
$X_{\dot{u}} = -142Kg.$	$Y_{\dot{v}} = -1715Kg.$
$N_{\dot{r}} = -1350N.m^2$	$X_{uu} = -35.4Kg.m^{-1}$
$X_{vv} = -128.4Kg.m^{-1}$	$Y_r = 435Kg.$
$Y_v = -346Kg.m^{-1}$	$Y_{v\|v\|} = -667Kg.m^{-1}$
$N_v = -686Kg.$	$N_{v\|v\|} = 443Kg.$
$N_r = -1427Kg.m.$	

TABLE II
THE INFANTE AUV: MODEL PARAMETERS

$k_1 = 1$	$k_2 = 1$	$k_3 = 1$	$k_4 = 1$	$k_5 = 1$
$k_\delta = 1$	$\psi_a = \pi/4$	$u_d = 1$	$\dot{u}_d = 0$	$\ddot{u}_d = 0$

TABLE III
CONTROLLER PARAMETERS

- Actuator saturation: in this study, the actuator saturation constraint has not been considered. This implies that our theoretical system is able to perfectly follow the path for any curvature. However, this is not true for a real system. In (Jiang and Nijmejer, 1999) and (Soetanto *et al.*, 2003) it is shown that a unicycle-type robot can achieve asymptotic convergence in a point-stabilization task, under saturation constraint. In the path-following situation, considering that the robot travels at a constant forward velocity, the problem is far more complex. Indeed, with the robot traveling at constant forward velocity, an asymptotic convergence requirement cannot be met on any path. One can argue that path design should take into account this constraint, thus respecting a 'followability' condition. However, an interesting question is the following : considering any path, what should the robot trajectory be to guarantee that the robot/path distance is minimal during the maneuver? This question can be elegantly posed using the idea of the virtual target, removing the constraint that this target is moving along the path. This adds another degree of freedom of the virtual controller that can be used to solve the problem.

B. Acceleration Measurement vs. Estimation

In this study, the dynamic model of the robot has been used to estimate the system accelerations. Nevertheless, the consideration of a dynamic model implies neglecting the unmodelled dynamics (high order terms, external disturbances, etc.). Measuring the acceleration captures all the phenomena that cause acceleration. But this kind of measurements id subject to noise interference, and the efficiency of their consideration should be proven. This is a tradeoff that should be explicitly studied. Consider the two following situations:

- Minor acceleration: in this situation, 3 solutions are possible:
  - use the dynamic model to estimate acceleration,
  - use acceleration measurements: this implies checking that the signal/noise ratio is sufficiently large to guarantee meaningful measurements,

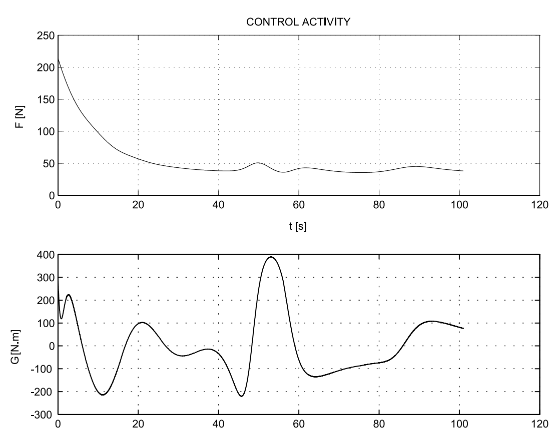

Fig. 6. Control Activity

  - ignore the acceleration.
- Major acceleration: in this situation, the acceleration must be explicitly taken into account. The efficiency of the 2 following solutions should therefor be studied:
  - use the dynamic model: this technique is efficient, especially if a robust adaptive scheme has been designed. Nevertheless, the high order term of the dynamic model and the external disturbances might not be negligible,
  - measure the acceleration: since the related variables are large, their measurement would seem appropriate. Moreover, the consideration of this measured variable simplifies the control expression.

C. Simplification of the control expression

These results need to be confirmed in tests under real conditions. A preliminary step consists in simplifying the control expression to obtain an easily implementable version, taking into consideration the sensors that the robot carries.

These questions are important and need to be explicitly studied.

VI. CONCLUSION

A nonlinear control law was developed for accurate path-following in autonomous underwater vehicles (AUVs). The key idea behind the new control law was to explicitly control the rate of progression of a 'virtual target' to be tracked along the path, thus overcoming the 'singularity' problems that arise when the position of the virtual target is simply defined by the projection of the actual vehicle onto that path. Controller design relies on backstepping techniques. The paper offered a formal proof of convergence of the vehicle's trajectory to the path. Simulation results illustrated the performance of the control system proposed.

The derived controller relies heavily on accurate knowledge of vehicle dynamics. Future work will address the problems of reducing controller complexity and evaluating its robustness against parameter uncertainty. The problem of precise path-following in the presence of unknown sea currents also warrants further consideration.

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