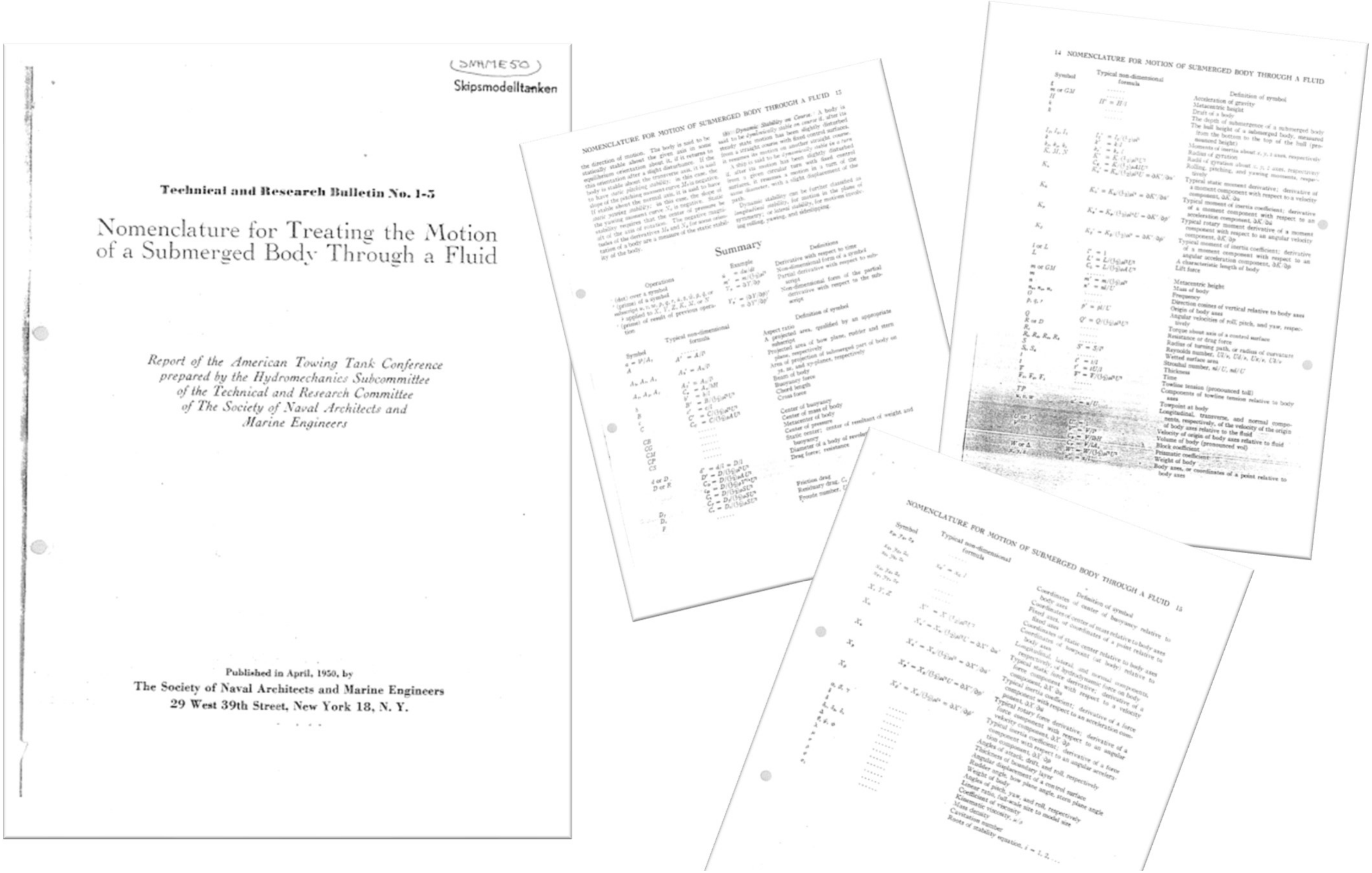
II - Models

Notation


(SNAME50)
Skipsmodelltanken
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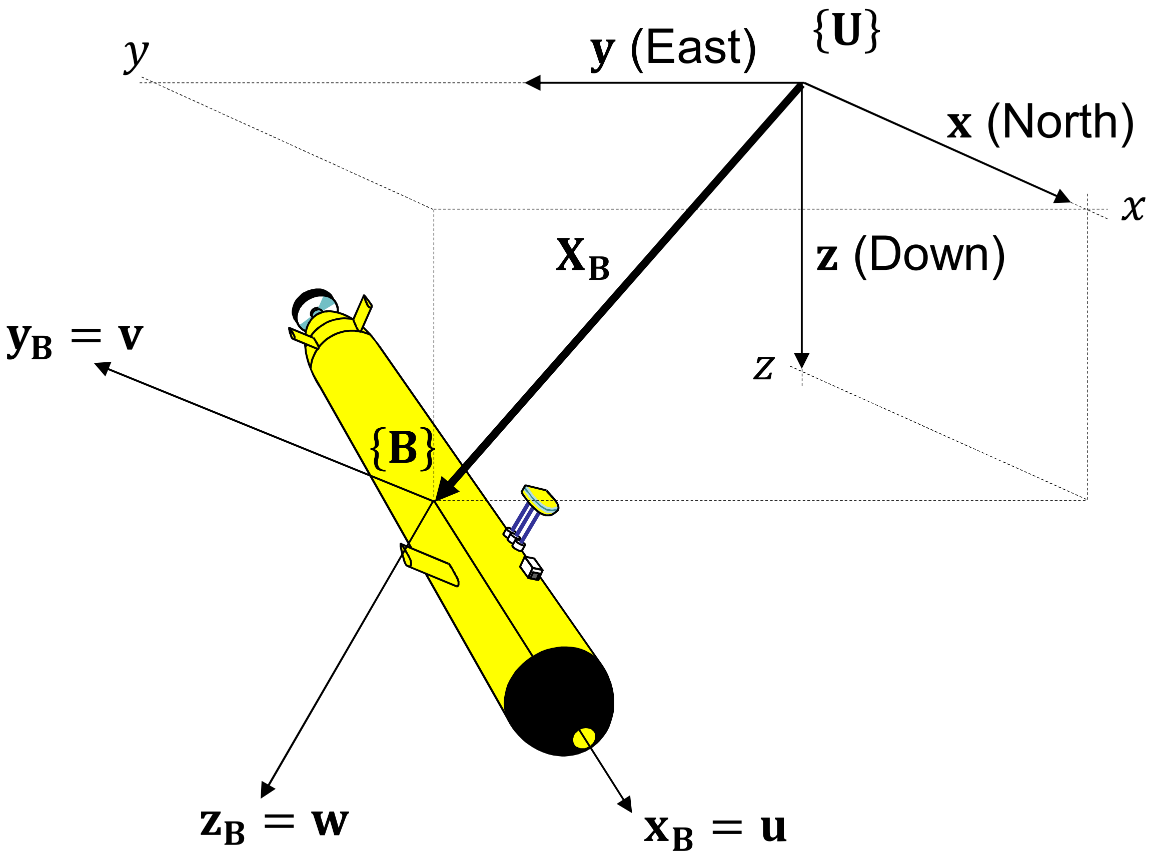
II – Models
a) Representation Formalism

Representation Formalism

- Position



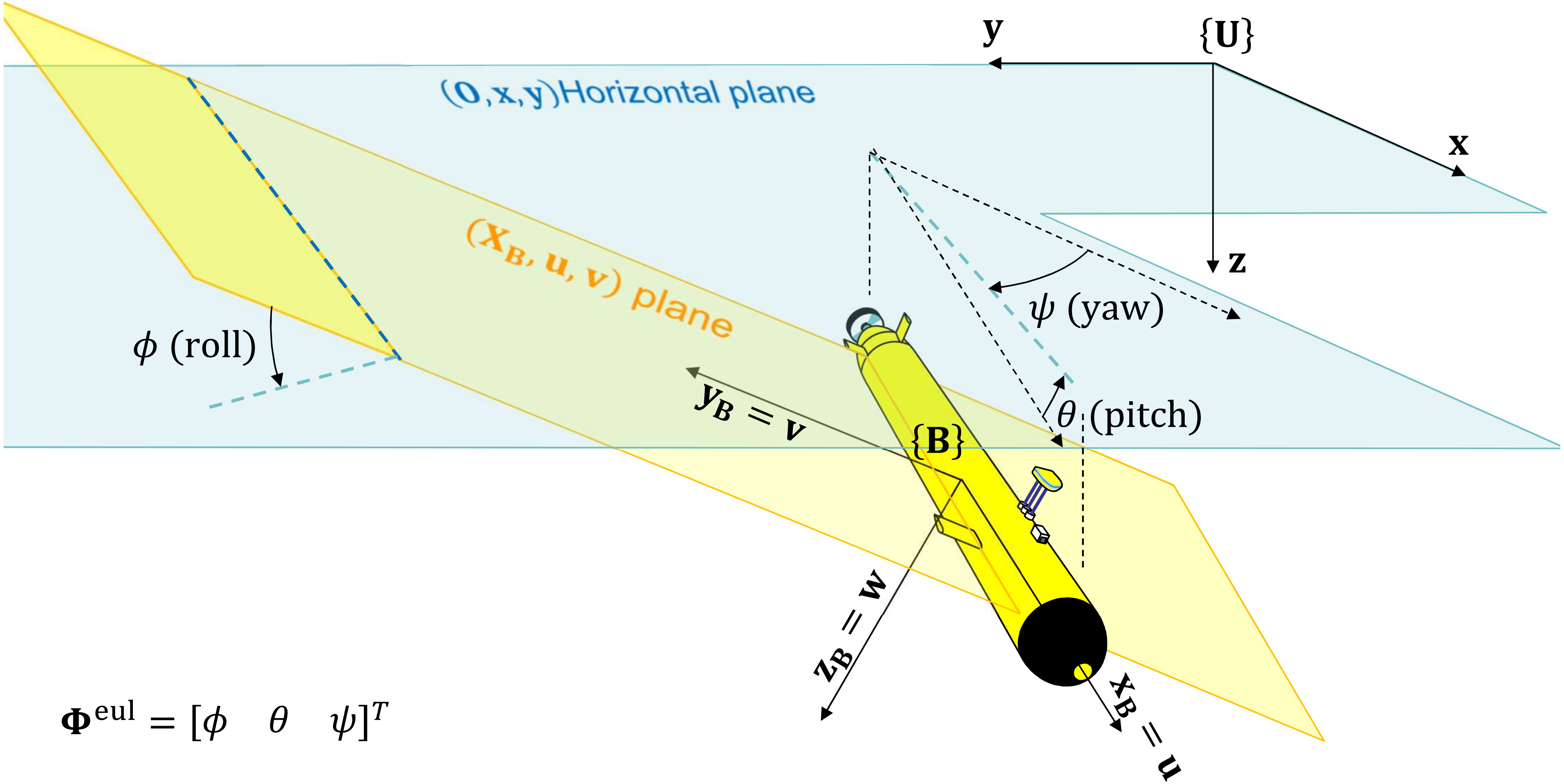
$\{\mathbf{U}\}: (\mathbf{O}, \mathbf{x}, \mathbf{y}, \mathbf{z})$

$\{\mathbf{B}\}: (\mathbf{X_B}, \mathbf{u}, \mathbf{v}, \mathbf{w})$

$\mathbf{X} = [x \quad y \quad z]^T$

Representation Formalism

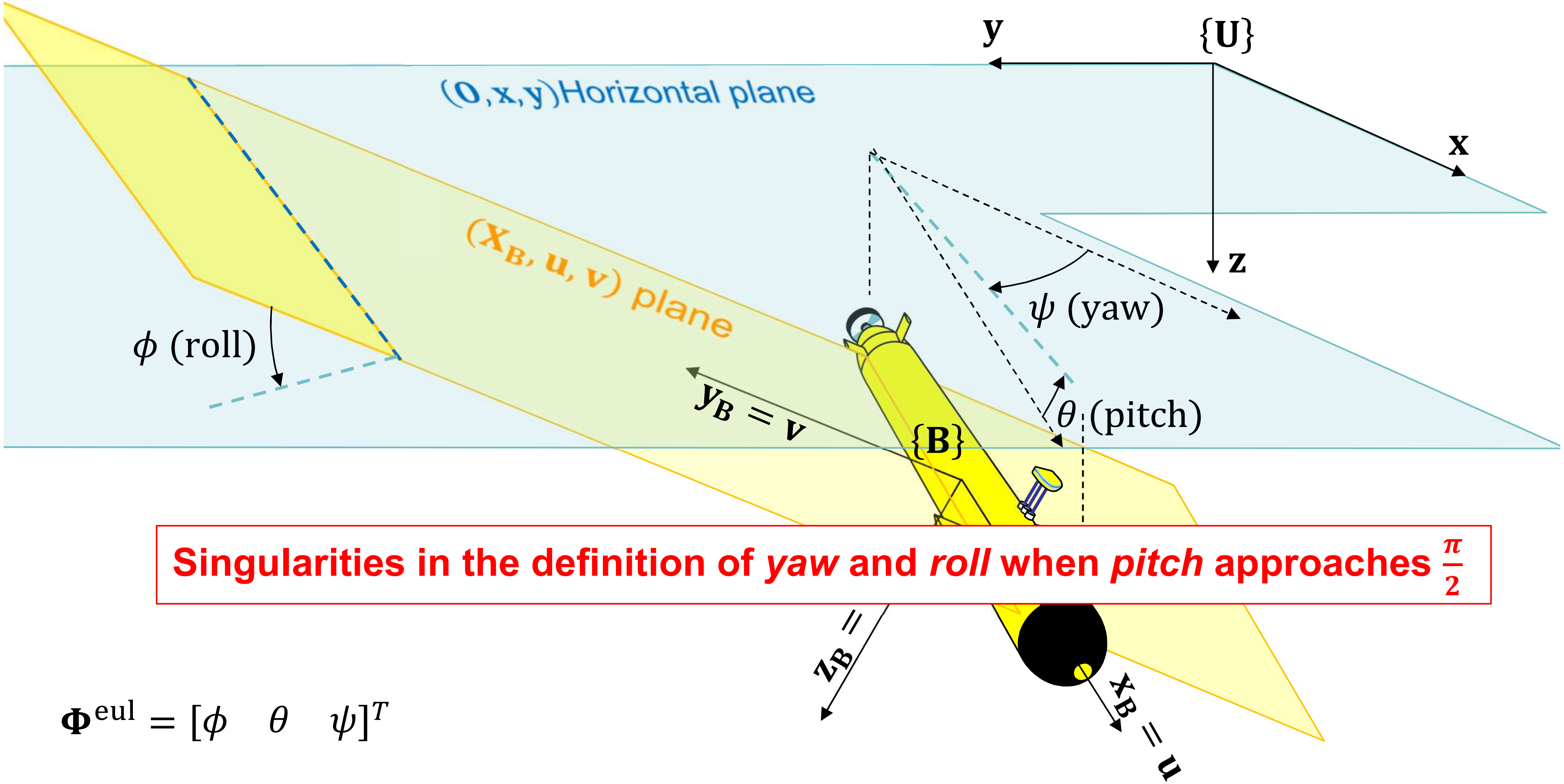
- Attitude, Orientation : Euler angles



$$\boldsymbol{\Phi}^{\text{eul}} = [\phi \quad \theta \quad \psi]^T$$

Representation Formalism

- Attitude, Orientation : Euler angles

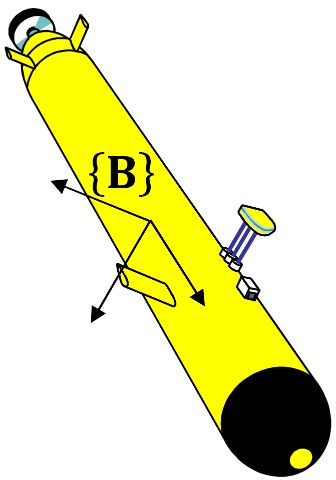


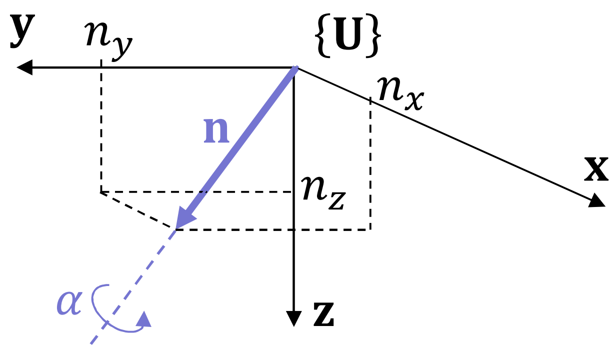
Singularities in the definition of *yaw* and *roll* when *pitch* approaches $\frac{\pi}{2}$

$$\boldsymbol{\Phi}^{\text{eul}} = [\phi \quad \theta \quad \psi]^T$$

Representation Formalism

- Attitude, Orientation : Rotation Matrix





$$\mathbf{n} = [n_x \quad n_y \quad n_z]^T, \|\mathbf{n}\| = 1$$

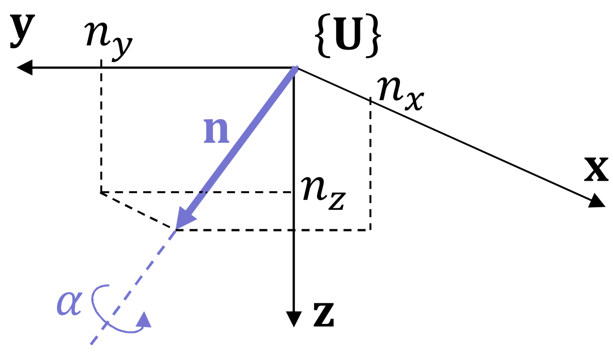
$$\mathbf{R} = \mathbf{P} + \cos\alpha \cdot (\mathbf{I} - \mathbf{P}) + \sin\alpha \cdot \mathbf{Q}$$

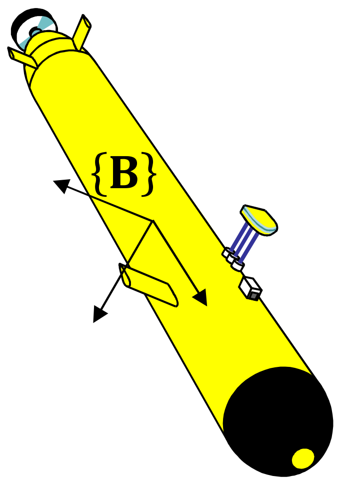
with : $\mathbf{P} = \mathbf{n} \cdot \mathbf{n}^T, \boldsymbol{Q} = \wedge(\mathbf{n})$

$$\mathbf{Q} = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}$$

Representation Formalism

- Attitude, Orientation : Quaternion





$$\mathbf{n} = [n_x \quad n_y \quad n_z]^T, \|\mathbf{n}\| = 1$$

$$\mathbf{Q}_B = \left[cos\left(\frac{\alpha}{2}\right) \quad \mathbf{n}^T \cdot sin\left(\frac{\alpha}{2}\right) \right]^T$$

$$\mathbf{Q}_B = [a \quad b \quad c \quad d]^T$$

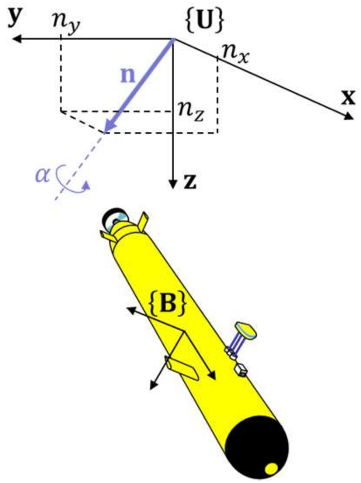
Representation Formalism

- Attitude, Orientation : Rot mat. vs Quaternion

$$\mathbf{R} = \mathbf{P} + \cos\alpha \cdot (\mathbf{I} - \mathbf{P}) + \sin\alpha \cdot \mathbf{Q}$$

with : $\mathbf{P} = \mathbf{n} \cdot \mathbf{n}^T, \boldsymbol{Q} = \wedge(\mathbf{n})$

$$\mathbf{Q} = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}$$



$$\mathbf{Q}_{\boldsymbol{B}} = \left[cos\left(\frac{\alpha}{2}\right) \quad \mathbf{n}^T \cdot sin\left(\frac{\alpha}{2}\right)\right]^T$$

$$\mathbf{Q}_{\boldsymbol{B}} = [a \quad b \quad c \quad d]^T$$

$$\mathbf{R} = \begin{pmatrix} a^2+b^2-c^2-d^2 & 2\cdot b\cdot c-2\cdot a\cdot d & 2\cdot a\cdot c+2\cdot b\cdot d \\ 2\cdot a\cdot d+2\cdot b\cdot c & a^2-b^2+c^2-d^2 & 2\cdot c\cdot d-2\cdot a\cdot b \\ 2\cdot b\cdot d-2\cdot a\cdot c & 2\cdot a\cdot b-2\cdot c\cdot d & a^2-b^2-c^2+d^2 \end{pmatrix}$$

$$r = \pm\frac{1}{2}\cdot\sqrt{1+\mathrm{Tr}(\mathbf{R})}; \begin{cases} a = r & c = \frac{1}{4\cdot r}\cdot(R_{13}-R_{31}) \\ b = \frac{1}{4\cdot r}\cdot(R_{32}-R_{23}) & d = \frac{1}{4\cdot r}\cdot(R_{21}-R_{12}) \end{cases}$$

Quaternion, basic relations

- Any rotation of an angle α around a unitary vector $\mathbf{n}$ can be expressed by the unitary quaternion :

$$\mathbf{Q} = \left[cos\left(\frac{\alpha}{2}\right) \quad \mathbf{n}^T \cdot sin\left(\frac{\alpha}{2}\right)\right]^T, \ \|\mathbf{n}\| = 1 \ \rightarrow \|\mathbf{Q}\| = 1$$

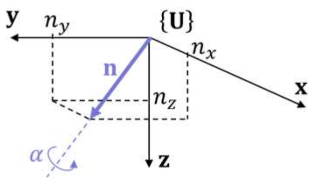
- The composition of 2 rotations, $\mathbf{Q_1}$ and $\mathbf{Q}_2$ can be expressed using the (non commutative) quaternionic multiplication, resulting in the $\mathbf{Q_3}$ quaternion such that :

$$\mathbf{Q}_3 = \mathbf{Q}_2 \otimes \mathbf{Q}_1$$

- The conjugate of a quaternion $\mathbf{Q}$ is denoted $\mathbf{Q}^*$ and exhibits the following properties:

$$\mathbf{Q}^* = \left[cos\left(\frac{\alpha}{2}\right) \quad -\mathbf{n}^T \cdot sin\left(\frac{\alpha}{2}\right)\right]^T;$$

$$(\mathbf{Q}_1 \otimes \mathbf{Q}_2)^* = \mathbf{Q}_2^* \otimes \mathbf{Q}_1^* \ ;$$



$$\mathbf{Q} \otimes \mathbf{Q}^* = \|\mathbf{Q}\| \cdot \mathbf{1_Q}, \text{ where } \mathbf{1_Q} = [\mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{0}]^{\boldsymbol{T}} \text{ is called } \textit{identity quaternion}$$

- A vector $\mathbf{v} \in \mathbb{R}^3$ can be expressed as a pure imaginary (non unitary) quaternion as:

$$\mathbf{V} = [0, \mathbf{v}^T]^T$$

- The rotation $\mathbf{Q}$ applied on a vector $\mathbf{v_1} \in \mathbb{R}^3$ results in a vector $\mathbf{v_2}$ expressed as:

$$\mathbf{V_2} = \mathbf{Q} \otimes \mathbf{V_1} \otimes \mathbf{Q}^*, \text{ where } \mathbf{V_1} = \left[0, \mathbf{v_1^T}\right]^T \text{ and } \ \mathbf{V_2} = \left[0, \mathbf{v_2^T}\right]^T$$

Quaternion, basic relations

- An object, onto which a frame $\{\mathbf{B}\}$: $(\mathbf{X_B}, \mathbf{u}, \mathbf{v}, \mathbf{w})$ is rigidly attached, in rotation w.r.t an intertial frame $\{\mathbf{U}\}$: $(\mathbf{O}, \mathbf{x}, \mathbf{y}, \mathbf{z})$, has an angular velocity vector denoted $\boldsymbol{\omega}$. The orientation of $\{\mathbf{B}\}$ w.r.t $\{\mathbf{U}\}$ is denoted with quaternion $\mathbf{Q}$. Hence the following relations hold :

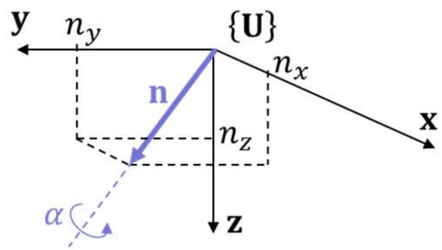
$$\boldsymbol{\Omega}_B = \mathbf{2} \cdot \mathbf{Q}^* \otimes \dot{\mathbf{Q}}, \text{ where } \boldsymbol{\Omega}_B = [0, \boldsymbol{\omega}_B^T]^T \text{ and } \boldsymbol{\omega}_B = [p, q, r]^{\boldsymbol{T}} \text{ is } \boldsymbol{\omega} \text{ expressed in } \{\mathbf{B}\}, \text{ and}$$

$$\boldsymbol{\Omega}_{\mathbf{0}} = \mathbf{2} \cdot \dot{\mathbf{Q}} \otimes \mathbf{Q}^*, \text{ where } \boldsymbol{\Omega}_0 = [0, \boldsymbol{\omega}_0^T]^T \text{ and } \boldsymbol{\omega}_0 \text{ is } \boldsymbol{\omega} \text{ expressed in } \{\mathbf{U}\}$$

- The left-multiplication of previous relation by $\mathbf{Q}$ yields the kinematic rotational model of the moving object as:

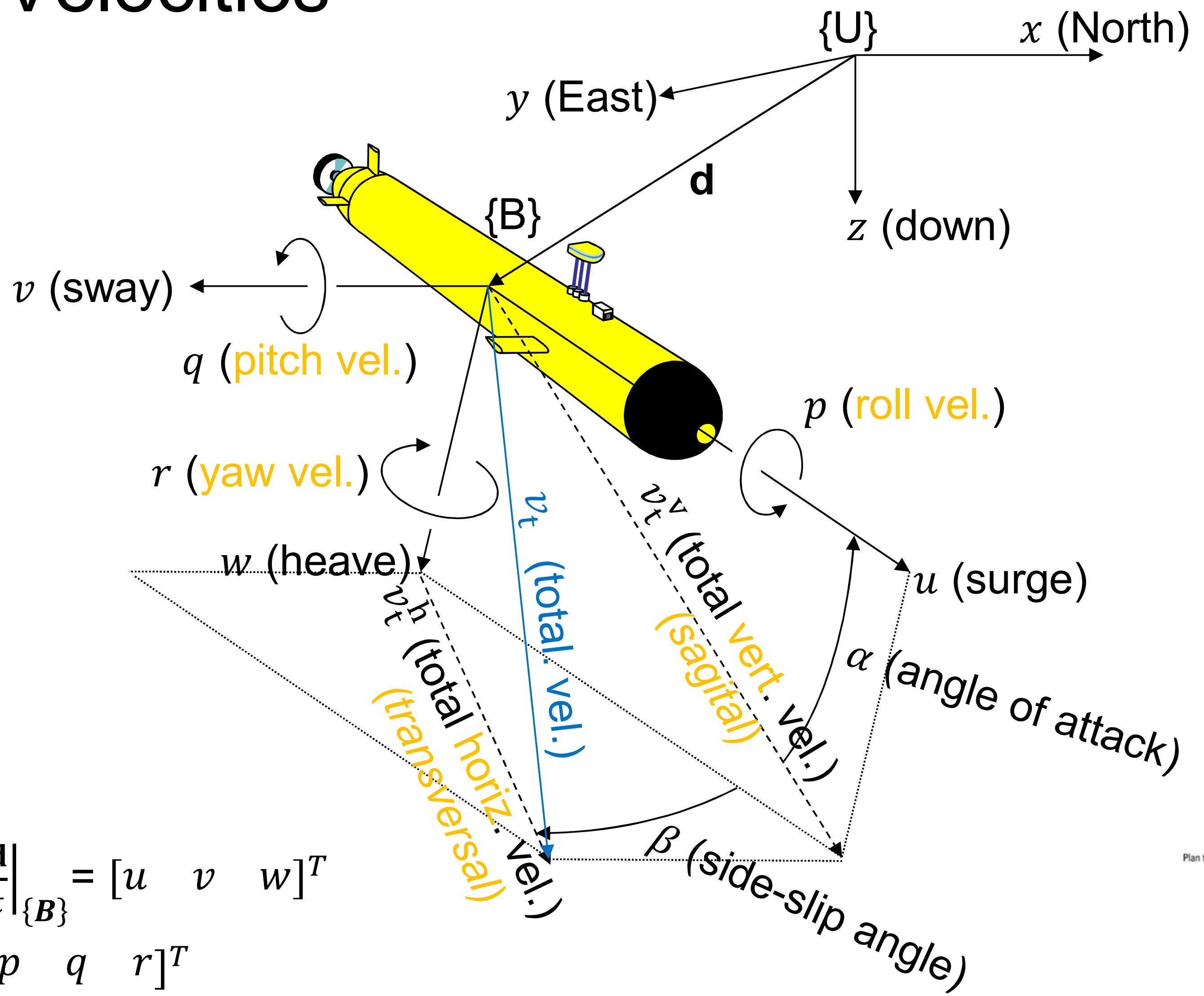
$$\dot{\mathbf{Q}} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{Q} \otimes \boldsymbol{\Omega}_B,$$

where $\boldsymbol{\Omega}_B = [0, \boldsymbol{\omega}_B^T]^T$, $\boldsymbol{\omega}_B = [p, q, r]^{\boldsymbol{T}}$
and p, q and r denote the object rotational velocities expressed in its own $\{\mathbf{B}\}$ frame, as described in the sequel.



Representation Formalism

- Velocities



$$\mathbf{v}_{\boldsymbol{B}} = \left.\frac{\mathrm{d}\mathbf{d}}{\mathrm{d}t}\right|_{\{\boldsymbol{B}\}} = [u \quad v \quad w]^T$$

$$\boldsymbol{\omega}_{\boldsymbol{B}} = [p \quad q \quad r]^T$$

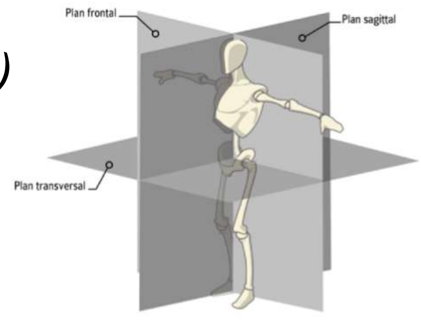


Figure 1 : Plans anatomiques (crédit photo sci-sport.com)

Representation Formalism

	Euler angles	Quaternions	Rotation Matrix
Position	$\mathbf{X} = [x \quad y \quad z]^T$		
Attitude	$\mathbf{\Phi}^{\mathrm{eul}} = [\phi \quad \theta \quad \psi]^T$	$\mathbf{Q}$	$\mathbf{R}$
Velocities	$\mathbf{V}_{\mathbf{B}} = [u \quad v \quad w]^T, \boldsymbol{\omega}_B = [p \quad q \quad r]^T$		
State	$\boldsymbol{\chi} = \begin{bmatrix} \boldsymbol{\eta} = \begin{bmatrix} \mathbf{X} \\ \mathbf{\Phi}^{\mathrm{eul}} \end{bmatrix} \\ \boldsymbol{\nu} = \begin{bmatrix} \mathbf{V}_{\mathbf{B}} \\ \boldsymbol{\omega}_B \end{bmatrix} \end{bmatrix}$	$\boldsymbol{\chi} = \begin{bmatrix} \boldsymbol{\eta} = \begin{bmatrix} \mathbf{X} \\ \boldsymbol{Q} \end{bmatrix} \\ \boldsymbol{\nu} = \begin{bmatrix} \mathbf{V}_{\mathbf{B}} \\ \boldsymbol{\omega}_B \end{bmatrix} \end{bmatrix}$	$\boldsymbol{\chi} = \{\mathbf{X}, \mathbf{R}, \mathbf{V}_B, \boldsymbol{\omega}_B\}$
Kinematic Model $\dot{\boldsymbol{\eta}} = \mathbf{f}(\boldsymbol{\nu})$	$\begin{bmatrix} \dot{\mathbf{X}} \\ \dot{\mathbf{\Phi}}^{\mathrm{eul}} \end{bmatrix} = \mathbf{R}_{\mathrm{cin}}^{\mathrm{eul}} \cdot \begin{bmatrix} \mathbf{V}_{\mathbf{B}} \\ \boldsymbol{\omega}_B \end{bmatrix}$	$\begin{cases} \dot{\mathbf{X}} = \mathbf{Q} \otimes [0, \mathbf{V}_B^T]^T \otimes \mathbf{Q}^* \\ \dot{\mathbf{Q}} = \frac{1}{2} \cdot \mathbf{Q} \otimes \underbrace{[0, \boldsymbol{\omega}_B^T]^T}_{\mathbf{\Omega}_B} \end{cases}$	$\begin{cases} \dot{\mathbf{X}} = \mathbf{R} \cdot \mathbf{V}_{\mathbf{B}} \\ \dot{\mathbf{R}} = \mathbf{R} \cdot (\boldsymbol{\omega}_{\mathbf{B}} \wedge) \end{cases}$

$$\mathbf{R}_{\mathrm{cin}}^{\mathrm{eul}} = \begin{bmatrix} \cos\psi \cdot \cos\theta & \cos\psi \cdot \sin\theta \cdot \sin\phi - \sin\psi \cdot \cos\phi & \cos\psi \cdot \sin\theta \cdot \cos\phi + \sin\psi \cdot \sin\phi & 0 & 0 & 0 \\ \sin\psi \cdot \cos\theta & \sin\psi \cdot \sin\theta \cdot \sin\phi + \cos\psi \cdot \cos\phi & \sin\psi \cdot \sin\theta \cdot \cos\phi - \cos\psi \cdot \sin\phi & 0 & 0 & 0 \\ -\sin\theta & \cos\theta \cdot \sin\phi & \cos\theta \cdot \cos\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin\phi \cdot \tan\theta & \cos\phi \cdot \tan\theta \\ 0 & 0 & 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & 0 & 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix}$$

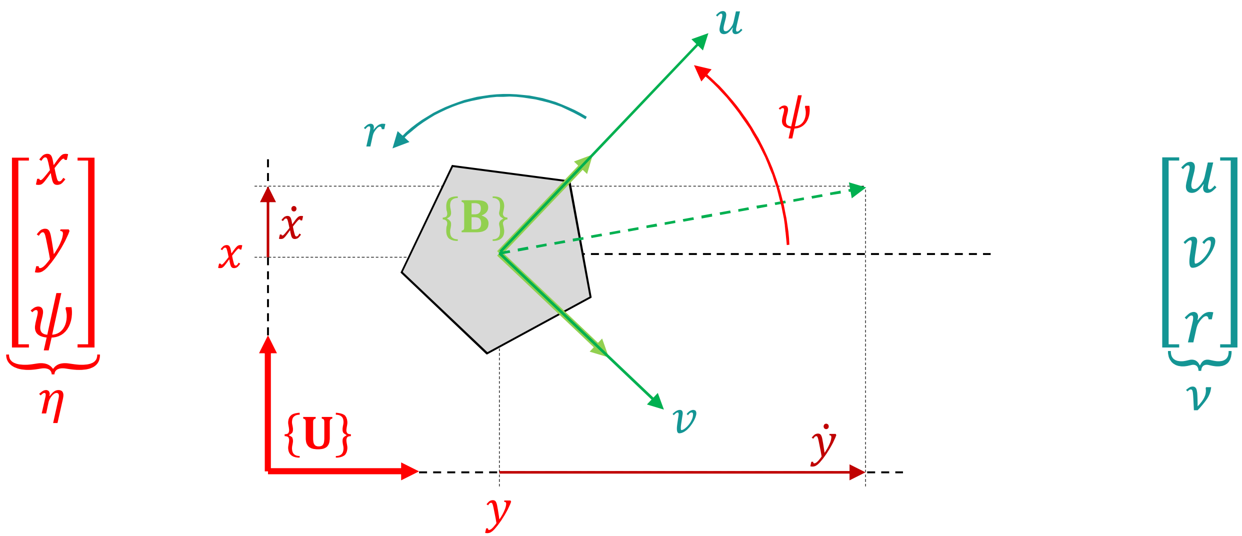
Gimbal lock, if $\theta = \pm\frac{\pi}{2}$

$$\boldsymbol{\omega}_B \wedge = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix}$$

II – Models
b) Kinematic Model

Kinematic Model

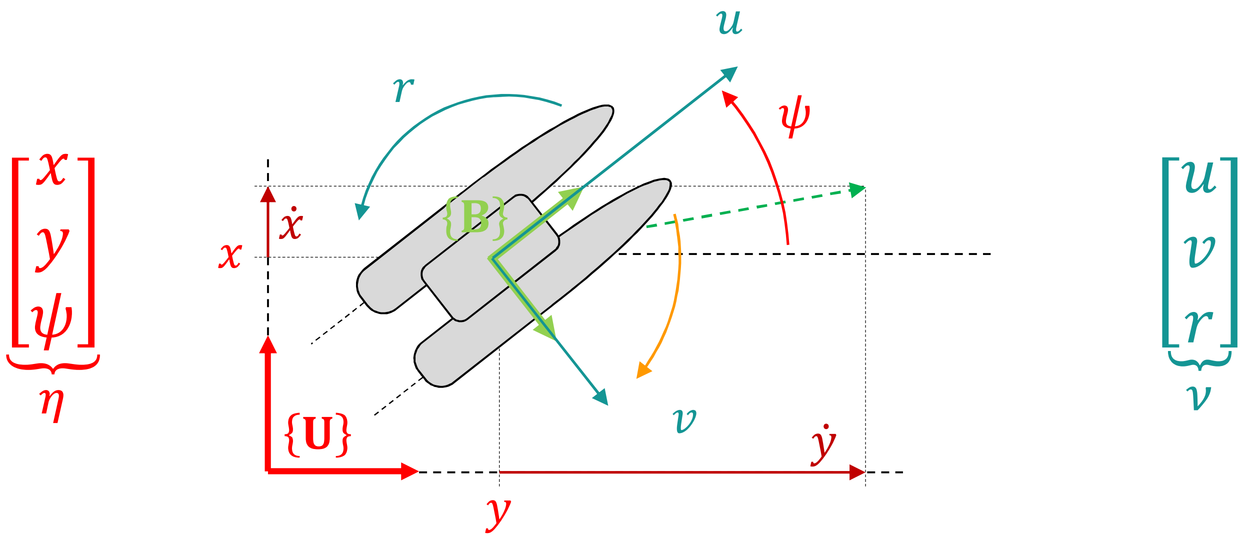
- 2D, Cartesian, no constraint



$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R}\cdot\boldsymbol{\nu}$$

Kinematic Model

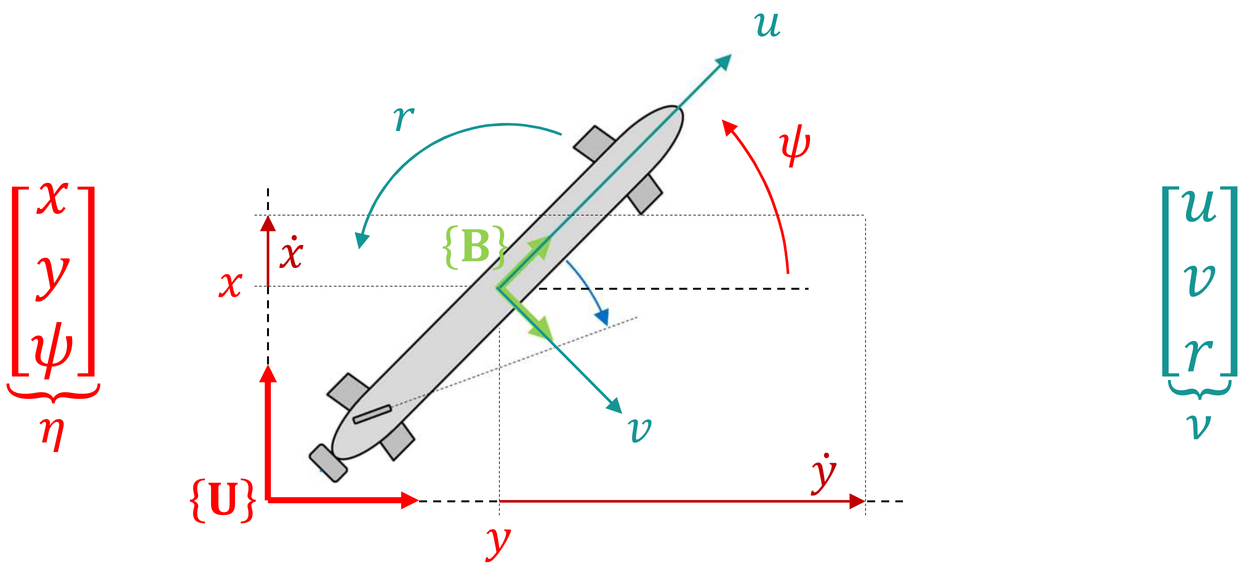
- 2D, Surface Craft, Cartesian, no constraint



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

Kinematic Model

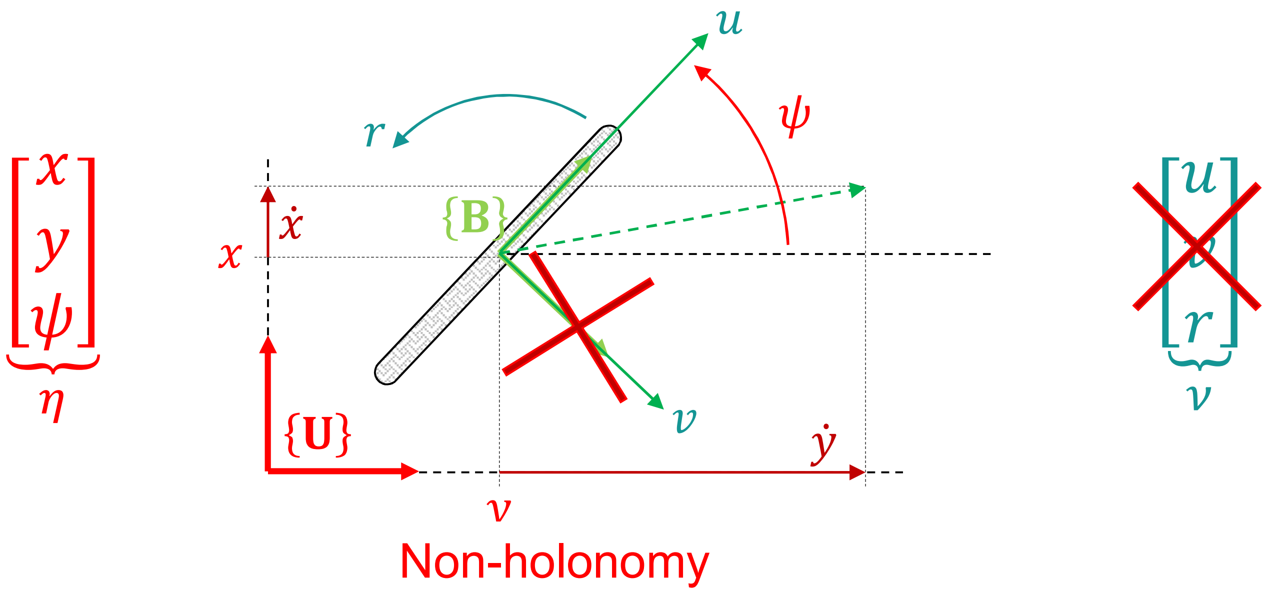
- 2D, Surface Craft, Cartesian, no constraint



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

Kinematic Model

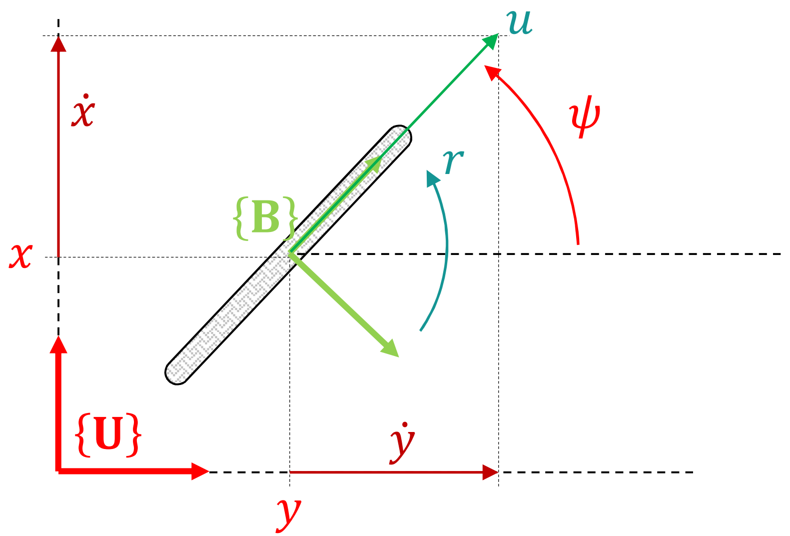
- 2D, Cartesian, the Wheel, nonholonomic constraint : $v = \dot{x} \cdot \cos\psi + \dot{y} \cdot \sin\psi = 0$



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

Kinematic Model

- 2D, Cartesian, the Wheel, nonholonomic constraint : $v = \dot{x} \cdot \cos\psi + \dot{y} \cdot \sin\psi = 0$

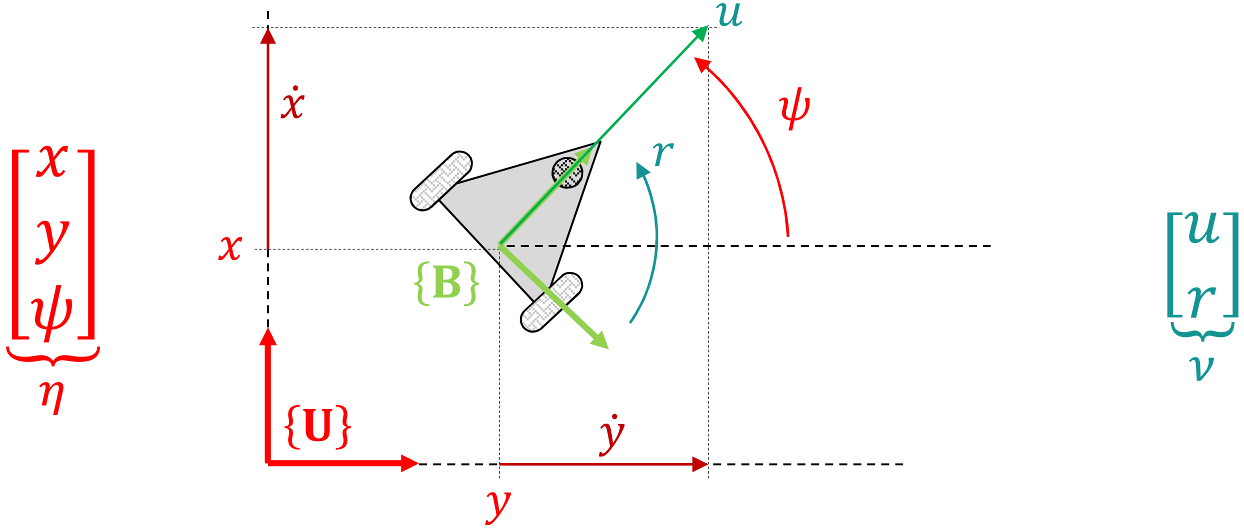


$$\underbrace{\begin{bmatrix} x \\ y \\ \psi \end{bmatrix}}_{\eta} \qquad \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\nu}$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{J} \cdot \boldsymbol{\nu}$$

Kinematic Model

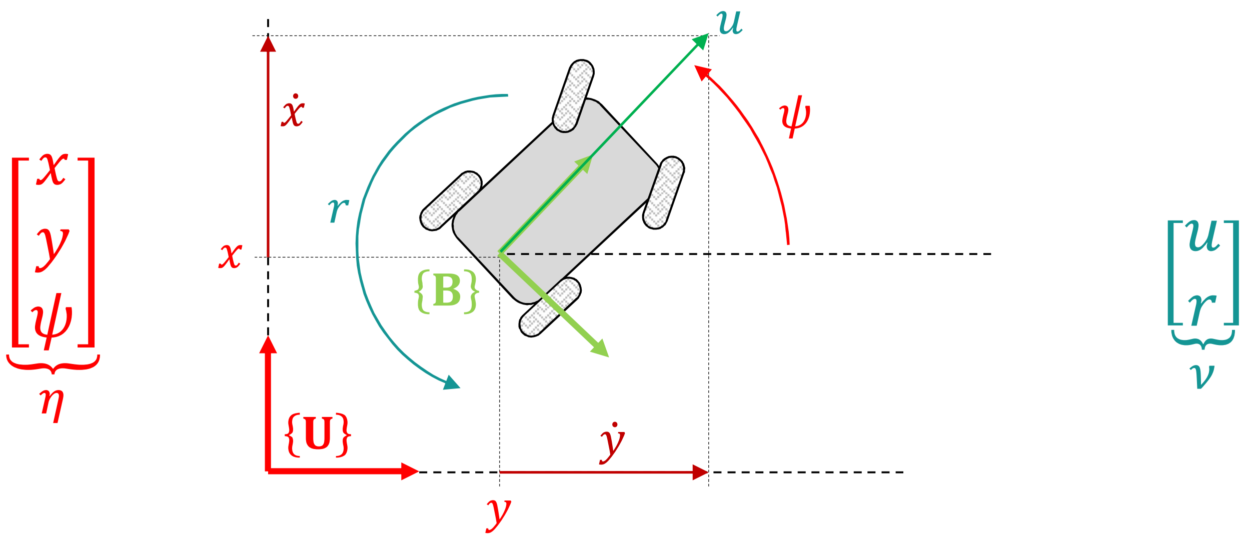
- 2D, Cartesian, the Unicycle, nonholonomic constraint : $v = \dot{x} \cdot \cos\psi + \dot{y} \cdot \sin\psi = 0$



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{J} \cdot \boldsymbol{\nu}$$

Kinematic Model

- 2D, Cartesian, the Car, nonholonomic constraint : $v = \dot{x} \cdot \cos\psi + \dot{y} \cdot \sin\psi = 0$

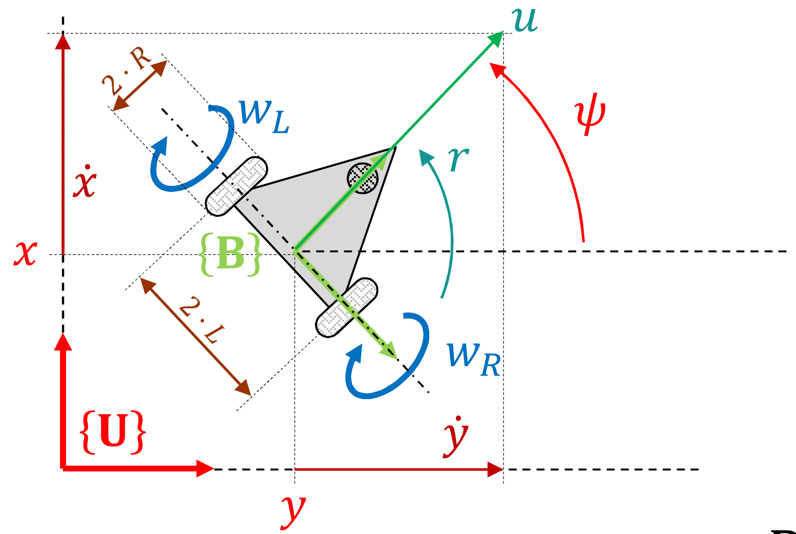


$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}} \rightarrow \dot{\boldsymbol{\eta}} = \mathbf{J} \cdot \boldsymbol{\nu}$$

II – Models
c) Actuation Model

Actuation Model

- 2D, the Unicycle



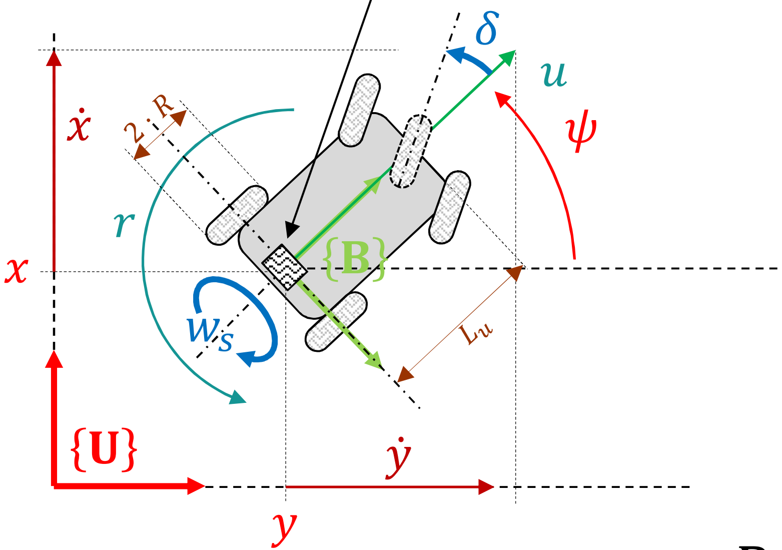
$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}} = \underbrace{\begin{bmatrix} \frac{R}{2} & \frac{R}{2} \\ \frac{R}{2 \cdot L} & \frac{-R}{2 \cdot L} \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} w_R \\ w_L \end{bmatrix}}_{\mathbf{U}} \rightarrow \boldsymbol{\nu} = \mathbf{A} \cdot \mathbf{U}$$

Actuation Model

- 2D, the Car

differential drive (K_R)

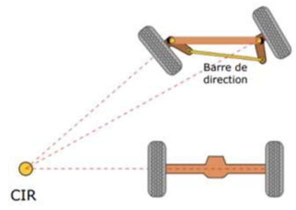


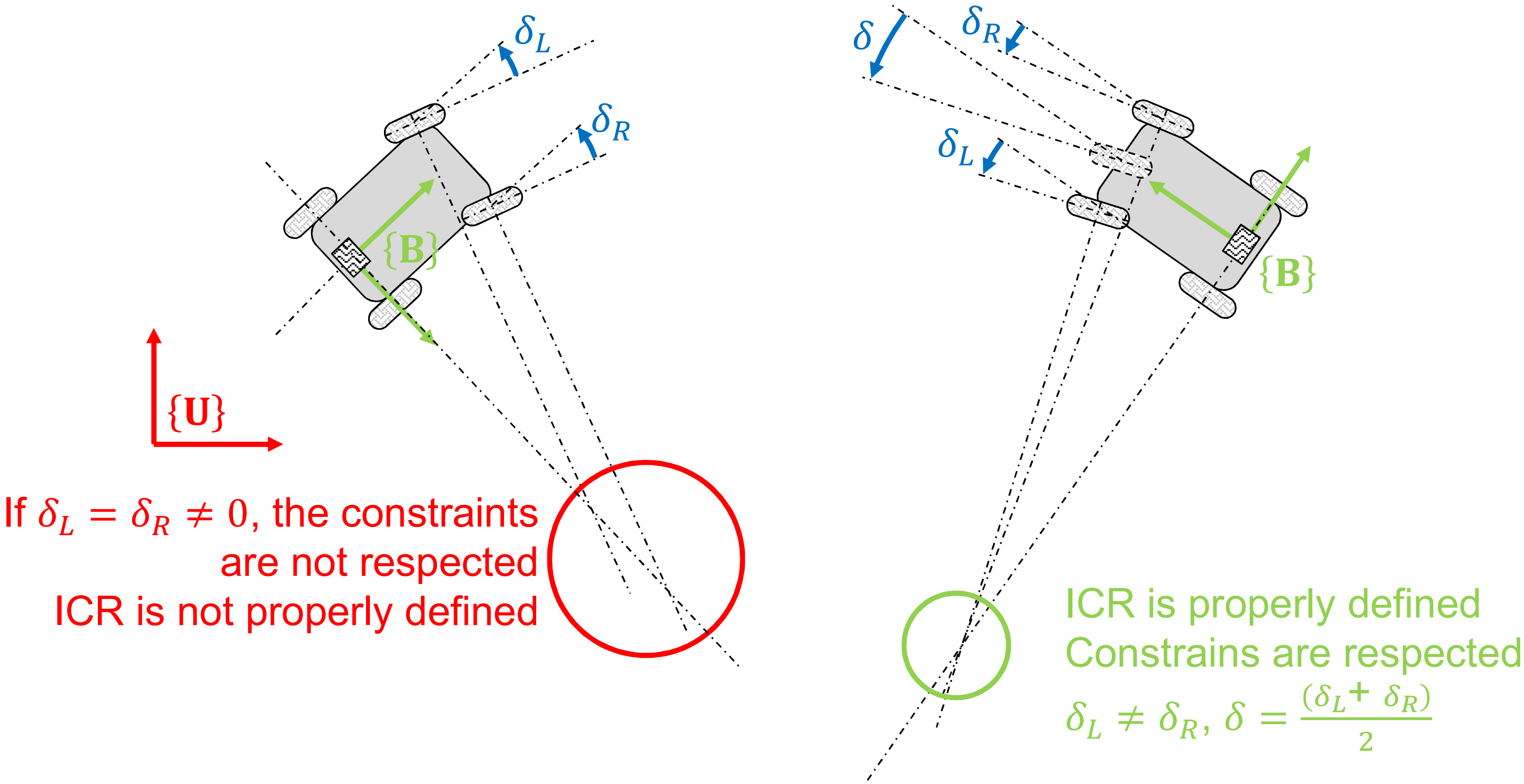
$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\underbrace{\begin{bmatrix} u \\ r \end{bmatrix}}_{\boldsymbol{\nu}} = \begin{bmatrix} \dfrac{R}{K_R} \cdot w_s \\ \dfrac{u \cdot \tan\delta}{L_u} \end{bmatrix} \rightarrow \boldsymbol{\nu} = \mathbf{A}(\mathbf{U})$$

Actuation Model

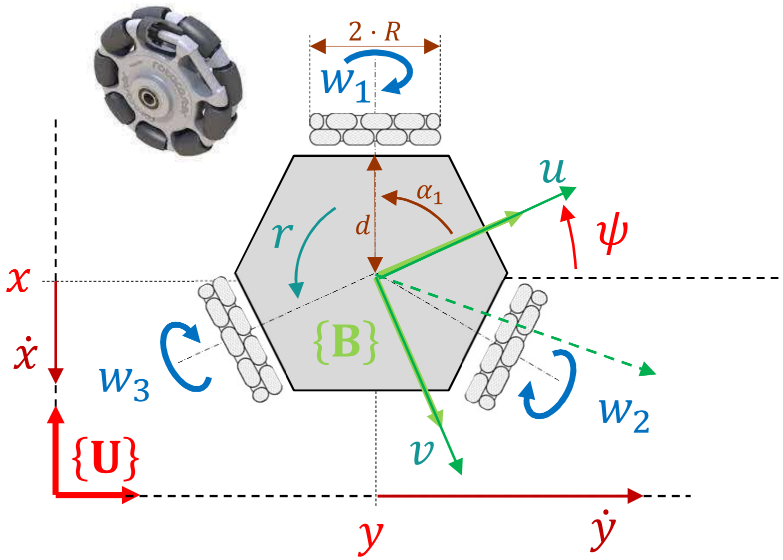
- 2D, the Car, non-sliding contraints





Actuation Model

- 2D, Cartesian, Omni-directional *sweedish wheels* system



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

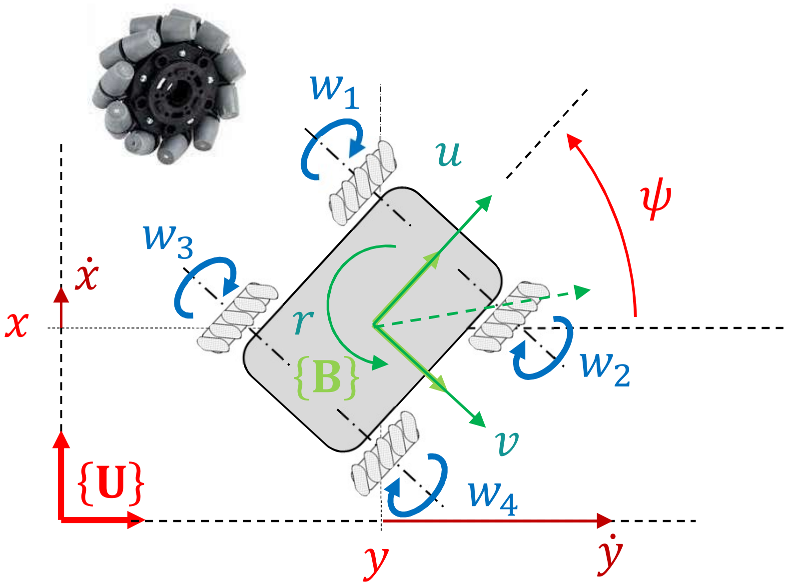
$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}}_{\mathbf{U}} = -\frac{1}{R} \cdot \underbrace{\begin{bmatrix} -\sin\alpha_1 & \cos\alpha_1 & d \\ -\sin\alpha_2 & \cos\alpha_2 & d \\ -\sin\alpha_3 & \cos\alpha_3 & d \end{bmatrix}}_{\mathbf{A}^{-1}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \boldsymbol{\nu} = \mathbf{A} \cdot \mathbf{U}$$

$$\alpha_1 = \frac{\pi}{3}, \alpha_2 = -\frac{\pi}{3}, \alpha_3 = \pi$$

Actuation Model

- 2D, Cartesian, Omni_directional *Mecanum wheels* system



$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

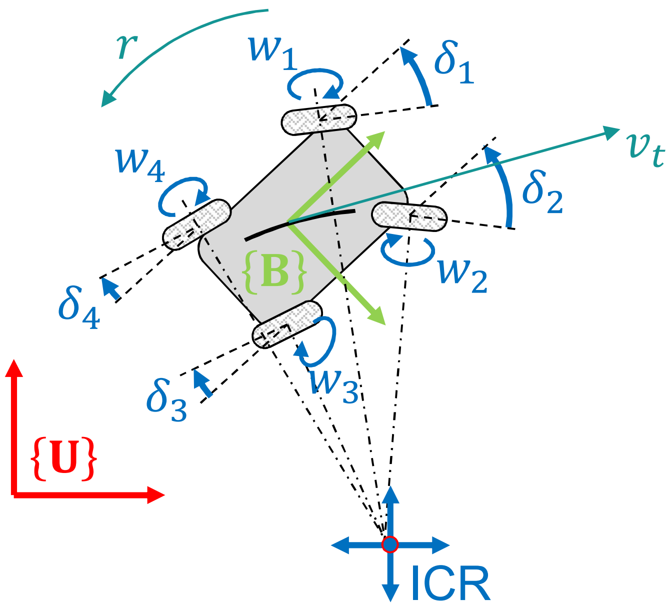
$$\underbrace{\begin{bmatrix}w_1\\ w_2\\ w_3\\ w_4\end{bmatrix}}_{\mathbf{U}} = \underbrace{\begin{bmatrix}1 & 1 & -(L_1+L_2)\\ -1 & 1 & -(L_1+L_2)\\ 1 & -1 & -(L_1+L_2)\\ -1 & -1 & -(L_1+L_2)\end{bmatrix}}_{\mathbf{A}^{-1}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

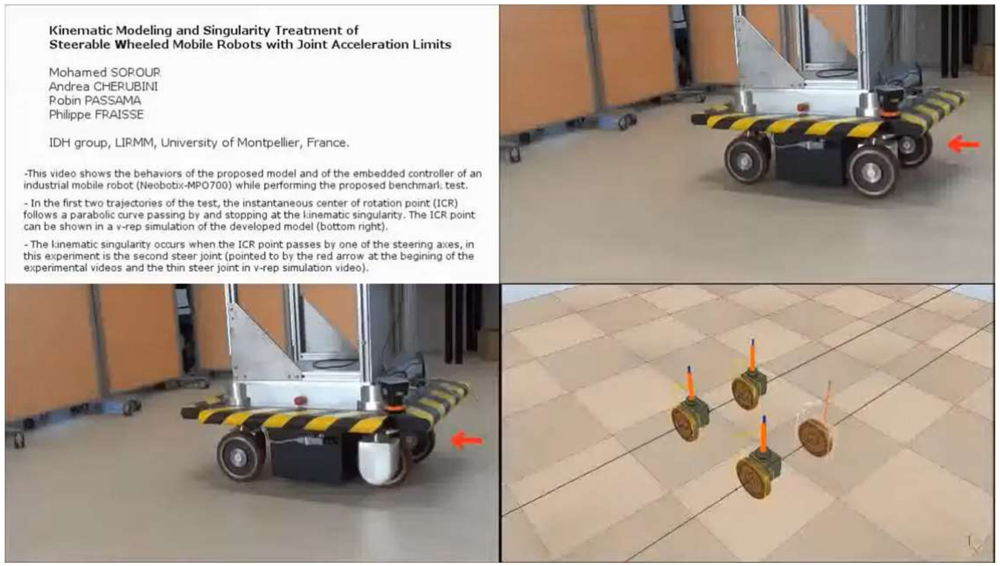
$$\rightarrow \boldsymbol{\nu} = \mathbf{A} \cdot \mathbf{U}$$

@ Forklift

Actuation Model

- 2D, the Car, Pseudo-Omni-Directional Wheeled Robots

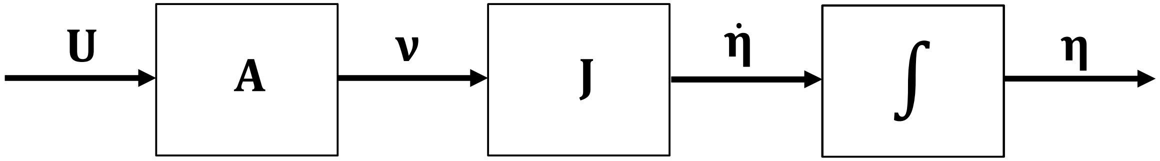




$$\begin{cases} \mathbf{W} = [w_1 \quad w_2 \quad w_3 \quad w_4]^T \\ \boldsymbol{\Delta} = [\delta_1 \quad \delta_2 \quad \delta_3 \quad \delta_4]^T \end{cases}, \begin{cases} \mathbf{W} = \mathbf{f}(v_t, r), \text{subject to } \Phi_{\mathbf{W}}(\mathbf{W}) = 0 \\ \boldsymbol{\Delta} = g(v_t, r), \text{subject to } \Phi_{\boldsymbol{\Delta}}(\boldsymbol{\Delta}) = 0 \end{cases}, \left\| \boldsymbol{X_{ICR}} \Big|_{\{\mathbf{B}\}} \right\|^{-1} = \frac{r}{v_t}$$

Simulation

- Kinematic simulation



Video sim Unicycle

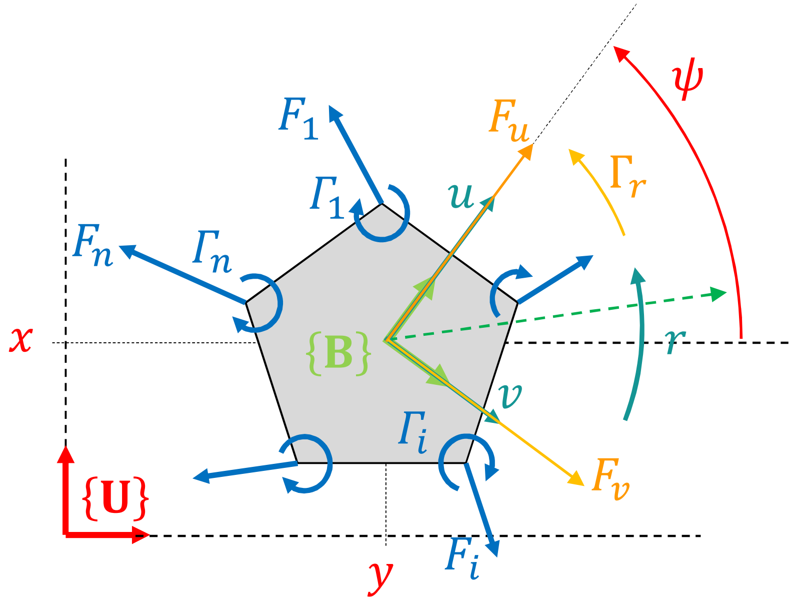
Video sim Omni 3 roues

Video sim Car

Video sim Omni Mecanum

Actuation Model

- 2D, Cartesian, Propulsive actuation



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R}(\boldsymbol{\eta}) \cdot \boldsymbol{\nu}$$

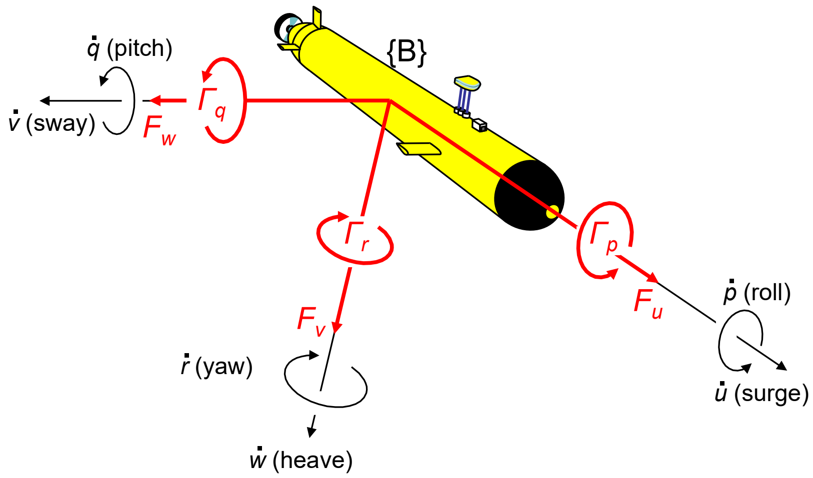
Dynamic Model:
$\dot{\boldsymbol{\nu}} = \mathbf{f}_{\mathrm{D}}(\boldsymbol{\Theta}, \boldsymbol{\eta}, \boldsymbol{\nu}, \mathbf{F}_B)$

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix}}_{\mathbf{F}_B} = \mathbf{A} \cdot \underbrace{[F_1, \Gamma_1, \dots, F_i, \Gamma_i, \dots, F_n, \Gamma_n]^T}_{\mathbf{F}_{\mathrm{m}}}$$

$$\rightarrow \mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_{\mathrm{m}}$$

Dynamic Model

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$



$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{\nu}}$$

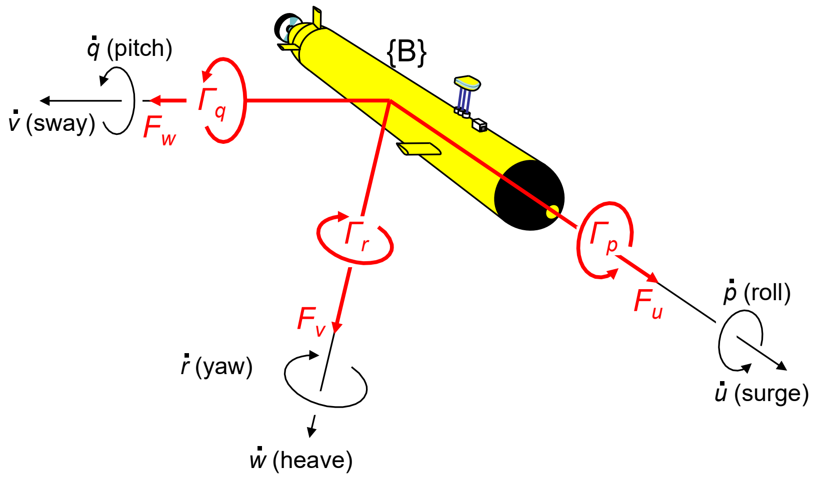
Hydro-dynamic phenomena (fluid/structure interaction) :

- Buoyancy
- Lift and Drag
- Added Mass

$$\mathbf{F}_B = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \dot{\boldsymbol{\nu}})$$

Dynamic Model

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$



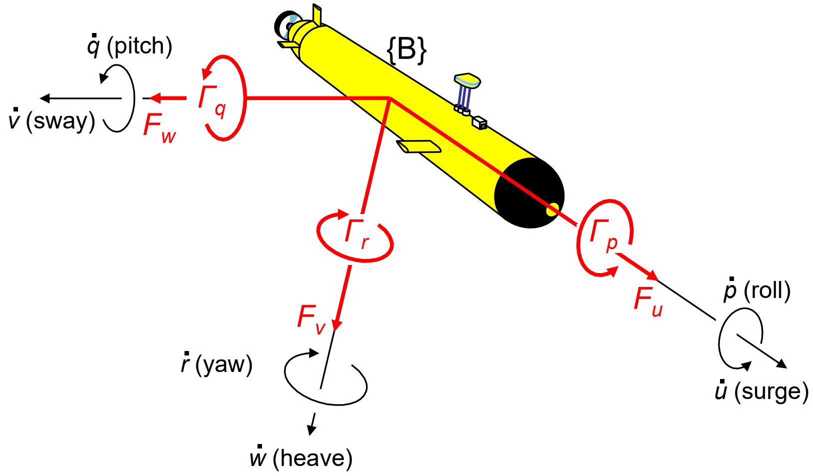
$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{\nu}}$$

$$\begin{aligned}
F_u &= X_{\dot{u}} \cdot \dot{u} + X_{u\cdot|u|} \cdot u \cdot |u| + X_{w\cdot q} \cdot w \cdot q + X_{v\cdot r} \cdot v \cdot r + X_G(\eta) \\
F_v &= Y_{\dot{v}} \cdot \dot{v} + Y_{v\cdot|v|} \cdot v \cdot |v| + Y_{u\cdot r} \cdot u \cdot r + Y_{w\cdot p} \cdot w \cdot p + Y_G(\eta) \\
F_w &= Z_{\dot{w}} \cdot \dot{w} + Z_{w\cdot|w|} \cdot w \cdot |w| + Z_{u\cdot q} \cdot u \cdot q + Z_{v\cdot p} \cdot v \cdot p + Z_G(\eta) \\
\Gamma_p &= K_{\dot{p}} \cdot \dot{p} + K_{p\cdot|p|} \cdot p \cdot |p| + K_{q\cdot r} \cdot q \cdot r + K_v \cdot v + K_w \cdot w + K_G(\eta) \\
\Gamma_q &= M_{\dot{q}} \cdot \dot{q} + M_{q\cdot|q|} \cdot q \cdot |q| + M_{p\cdot r} \cdot p \cdot r + M_u \cdot u + M_w \cdot w + M_G(\eta) \\
\Gamma_r &= N_{\dot{r}} \cdot \dot{r} + N_{r\cdot|r|} \cdot r \cdot |r| + N_{p\cdot q} \cdot p \cdot q + N_u \cdot u + N_v \cdot v + N_G(\eta)
\end{aligned}$$

$$\mathbf{F}_B = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \dot{\boldsymbol{\nu}})$$

Dynamic Model

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$



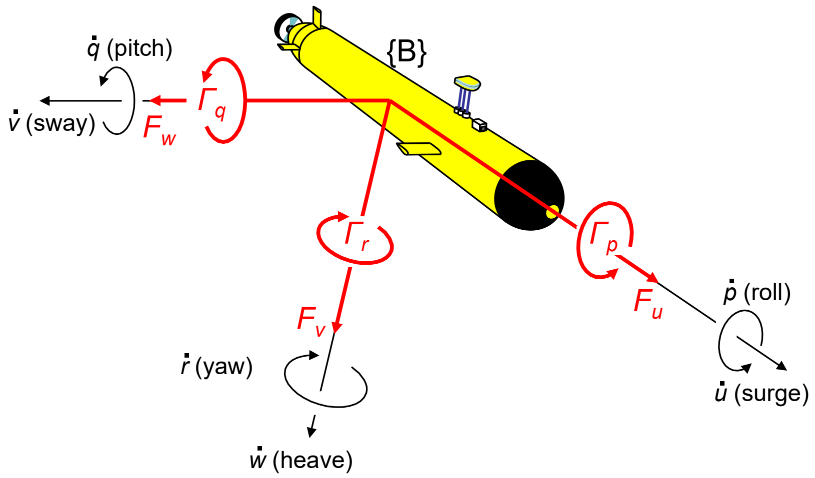
$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{\nu}}$$

$$\begin{aligned}
F_u &= X_{\dot{u}} \cdot \dot{u} + X_{u\cdot|u|} \cdot u \cdot |u| + X_{w\cdot q} \cdot w \cdot q + X_{v\cdot r} \cdot v \cdot r + X_G(\eta) \\
F_v &= Y_{\dot{v}} \cdot \dot{v} + Y_{v\cdot|v|} \cdot v \cdot |v| + Y_{u\cdot r} \cdot u \cdot r + Y_{w\cdot p} \cdot w \cdot p + Y_G(\eta) \\
F_w &= Z_{\dot{w}} \cdot \dot{w} + Z_{w\cdot|w|} \cdot w \cdot |w| + Z_{u\cdot q} \cdot u \cdot q + Z_{v\cdot p} \cdot v \cdot p + Z_G(\eta) \\
\Gamma_p &= K_{\dot{p}} \cdot \dot{p} + K_{p\cdot|p|} \cdot p \cdot |p| + K_{q\cdot r} \cdot q \cdot r + K_v \cdot v + K_w \cdot w + K_G(\eta) \\
\Gamma_q &= M_{\dot{q}} \cdot \dot{q} + M_{q\cdot|q|} \cdot q \cdot |q| + M_{p\cdot r} \cdot p \cdot r + M_u \cdot u + M_w \cdot w + M_G(\eta) \\
\Gamma_r &= N_{\dot{r}} \cdot \dot{r} + N_{r\cdot|r|} \cdot r \cdot |r| + N_{p\cdot q} \cdot p \cdot q + N_u \cdot u + N_v \cdot v + N_G(\eta)
\end{aligned}$$

$$\mathbf{F}_B = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \dot{\boldsymbol{\nu}})$$

Dynamic Model

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$



$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{\nu}}$$

A NONLINEAR UNIFIED STATE-SPACE MODEL FOR SHIP MANEUVERING AND CONTROL IN A SEAWAY

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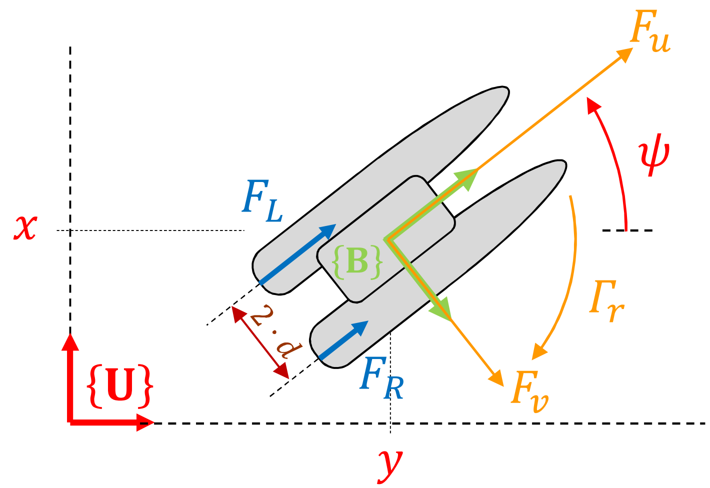
This article presents a unified state-space model for ship maneuvering, station-keeping, and control in a seaway. The frequency-dependent potential and viscous damping terms, which in classic theory results in a convolution integral not suited for real-time simulation, is compactly represented by using a state-space formulation. The separation of the vessel model into a low-frequency model (represented by zero-frequency added mass and damping) and a wave-frequency model (represented by motion transfer functions or RAOs), which is commonly used for simulation, is hence made superfluous.

Keywords: ship modelling, equations of motion, hydrodynamics, maneuvering, seakeeping, autopilots, dynamic positioning.

Actuation Model

- 2D, Cartesian, Propulsive actuation, ASV



$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix}}_{\mathbf{F}_B} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ d & -d \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} F_L \\ F_R \end{bmatrix}^T}_{\mathbf{F}_m}$$

$$\rightarrow \mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

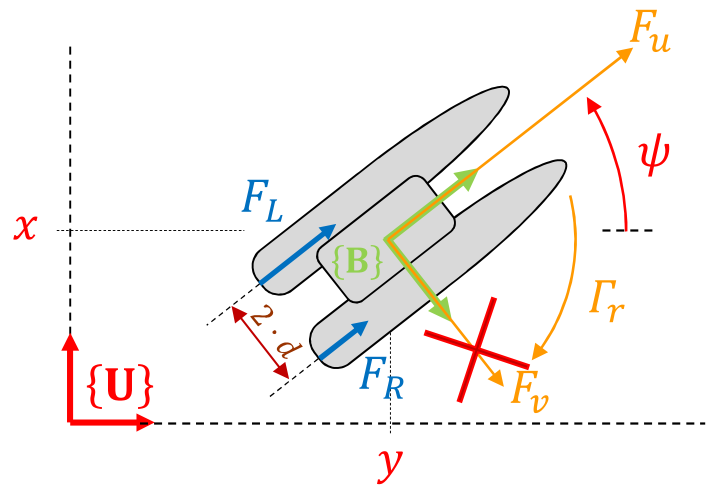
$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}}_{\dot{\boldsymbol{\nu}}} = \begin{cases} \dfrac{1}{X_{\dot{u}}} \cdot (F_u - X_u \cdot u \cdot |u|) \\ \dfrac{1}{Y_{\dot{v}}} \cdot (F_v - Y_v \cdot v \cdot |v| - Y_{ur} \cdot u \cdot r) \\ \dfrac{1}{N_{\dot{u}}} \cdot (\Gamma_r - N_r \cdot r \cdot |r|) \end{cases}$$

$$\rightarrow \dot{\boldsymbol{\nu}} = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \mathbf{F}_B)$$

Actuation Model

- 2D, Cartesian, Propulsive actuation, ASV



$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix}}_{\mathbf{F}_B} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ d & -d \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} F_L \\ F_R \end{bmatrix}^T}_{\mathbf{F}_m}$$

$$\rightarrow \mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

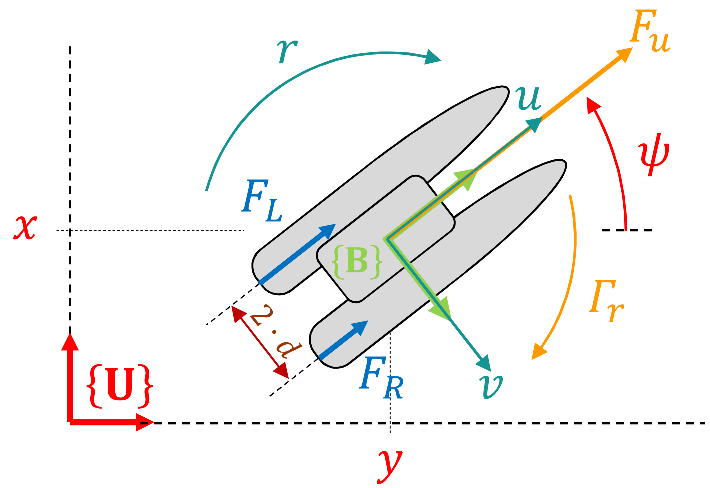
$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}}_{\dot{\boldsymbol{\nu}}} = \begin{cases} \frac{1}{X_{\dot{u}}} \cdot (F_u - X_u \cdot u \cdot |u|) \\ \frac{1}{Y_{\dot{v}}} \cdot (\Gamma_v - Y_v \cdot v \cdot |v| - Y_{ur} \cdot u \cdot r) \\ \frac{1}{N_{\dot{u}}} \cdot (\Gamma_r - N_r \cdot r \cdot |r|) \end{cases}$$

$$\rightarrow \dot{\boldsymbol{\nu}} = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \mathbf{F}_B)$$

Under-actuation

Actuation Model

- 2D, Cartesian, Propulsive actuation, ASV



$$\underbrace{\begin{bmatrix}F_u\\ \Gamma_r\end{bmatrix}}_{\mathbf{F}_B} = \underbrace{\begin{bmatrix}1 & 1\\ d & -d\end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix}F_L\\ F_R\end{bmatrix}^T}_{\mathbf{F}_m}$$

$$\rightarrow \mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

$$\underbrace{\begin{bmatrix}\dot x\\ \dot y\\ \dot\psi\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

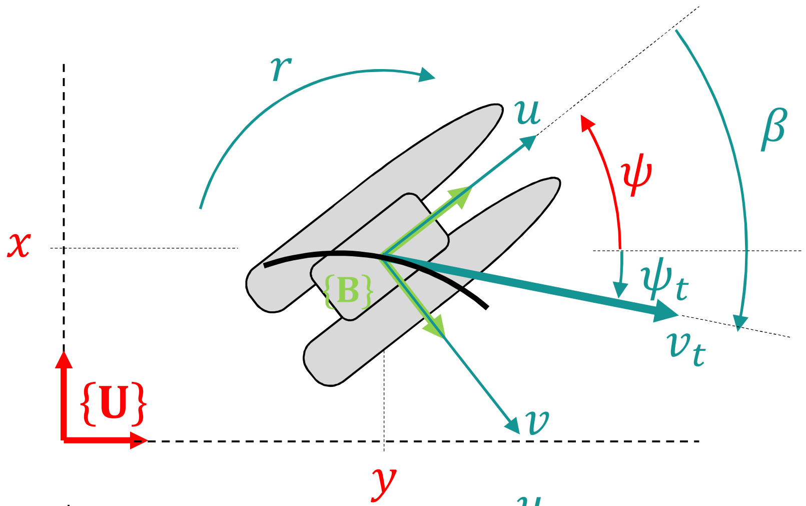
$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R} \cdot \boldsymbol{\nu}$$

$$\underbrace{\begin{bmatrix}\dot u\\ \dot v\\ \dot r\end{bmatrix}}_{\dot{\boldsymbol{\nu}}} = \begin{cases} \frac{1}{X_{\dot u}} \cdot (F_u - X_u \cdot u \cdot |u|) \\ \frac{1}{Y_{\dot v}} \cdot (-Y_v \cdot v \cdot |v| - Y_{ur} \cdot u \cdot r) \\ \frac{1}{N_{\dot u}} \cdot (\Gamma_r - N_r \cdot r \cdot |r|) \end{cases}$$

$$\rightarrow \dot{\boldsymbol{\nu}} = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \mathbf{F}_B)$$

Back to kinematics

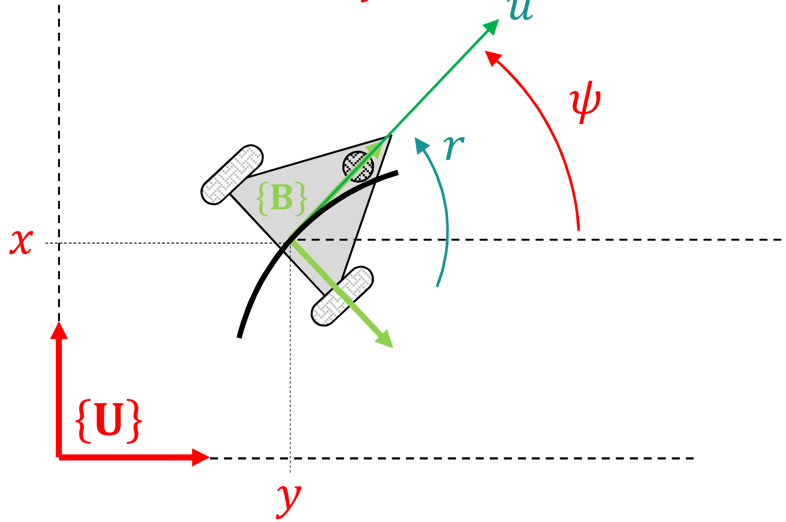
- 2D, Cartesian, ASV vs Unicycle



$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

$$= \begin{bmatrix}\cos\psi_t & 0\\ \sin\psi_t & 0\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}v_t\\ \dot{\psi}_t\end{bmatrix}$$

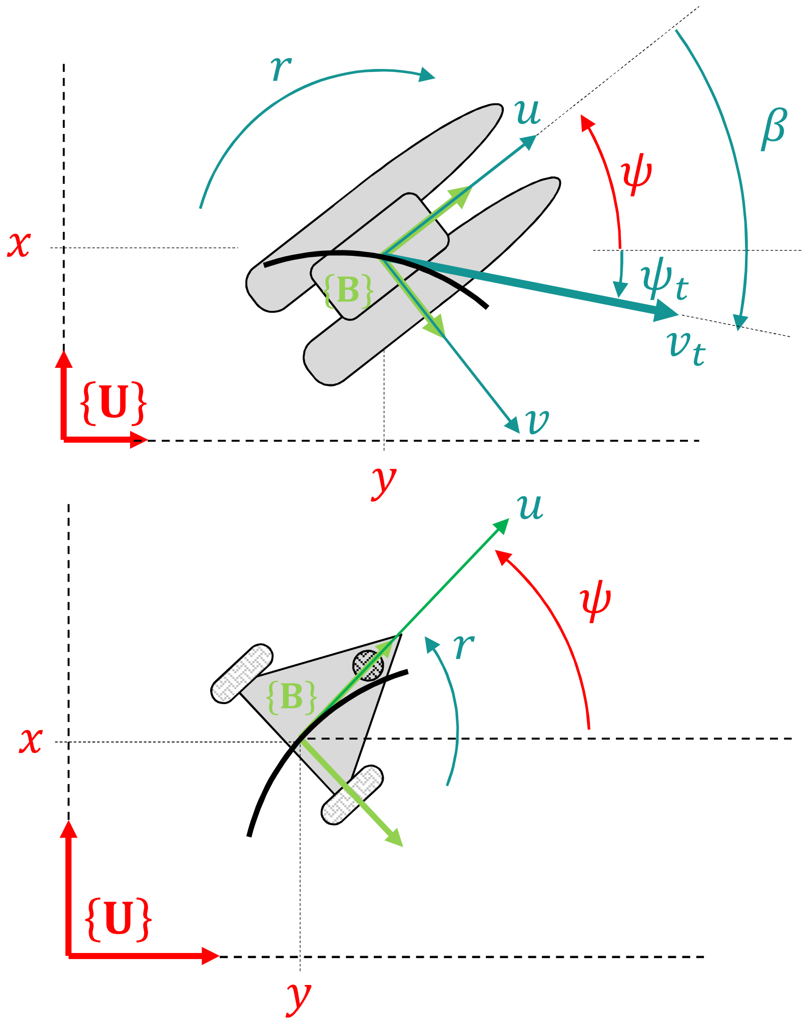
The total velocity of a moving object is necessarily tangent to its own trajectory



$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & 0\\ \sin\psi & 0\\ 0 & 1\end{bmatrix}}_{\mathbf{J}} \cdot \underbrace{\begin{bmatrix}u\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

Back to kinematics

- 2D, Cartesian, ASV vs Unicycle



$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} == \begin{bmatrix} \cos\psi_t & 0 \\ \sin\psi_t & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_t \\ \dot{\psi}_t \end{bmatrix}$$

$$\dot{\psi}_t = r + \dot{\beta}, \text{where } \beta = \operatorname{atan}\frac{v}{u}$$

$$\begin{cases} F_u = m_u \cdot \dot{u} + d_u \\ 0 = m_v \cdot \dot{v} + m_{ur} \cdot u \cdot r + d_v \\ \Gamma_r = m_r \cdot \dot{r} + d_r \end{cases}$$

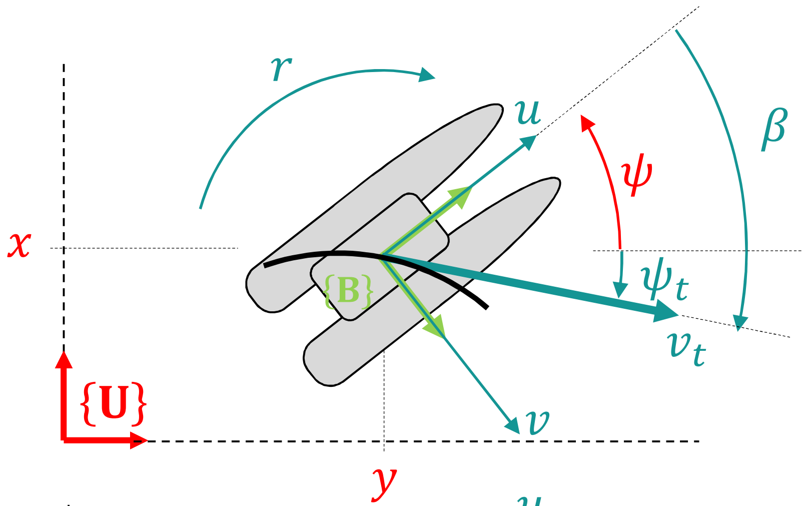
$$\rightarrow r = \frac{\dot{\psi}_t + f(\beta, \mathbf{v}, \dot{u})}{1 - \cos^2\beta \cdot \dfrac{m_{ur}}{m_v}}$$

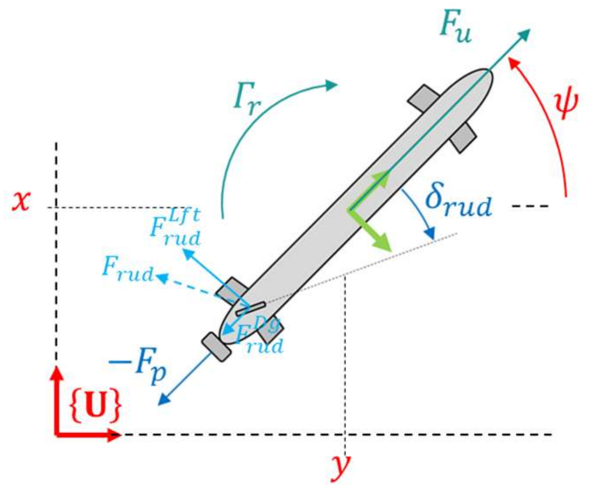
$$\text{well posed if } \frac{m_{ur}}{m_v} = \frac{m - Y_r}{m - Y_{\dot{v}}} < 1$$

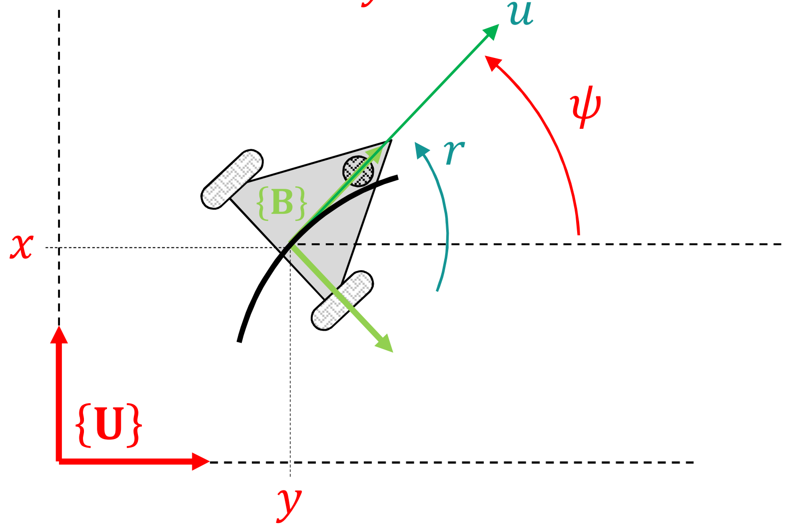
Covered by *stern dominancy* assumption
(open-loop local stability)

Back to kinematics

- 2D, Cartesian, ASV, AUV vs Unicycle





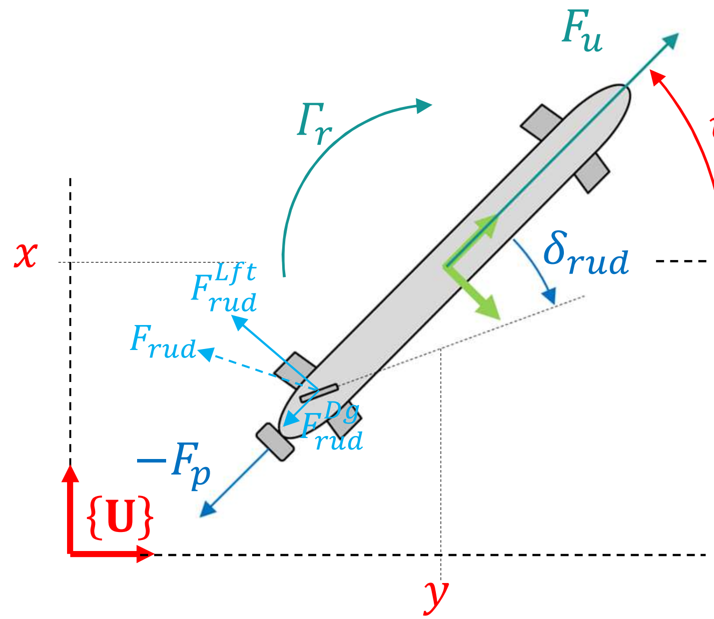


$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}}_{\dot{\boldsymbol{\eta}}} == \begin{bmatrix} \cos\psi_t & 0 \\ \sin\psi_t & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_t \\ r + \dot{\beta} \end{bmatrix}$$

Covered by *stern dominancy* assumption
(open-loop local stability)

Actuation Model

- 2D, Cartesian, Propulsive actuation, AUV



$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}}_{\dot{\boldsymbol{\eta}}} = \underbrace{\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{bmatrix}u\\ v\\ r\end{bmatrix}}_{\boldsymbol{\nu}}$$

$$\rightarrow \dot{\boldsymbol{\eta}} = \mathbf{R}\cdot\boldsymbol{\nu}$$

$$\underbrace{\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{bmatrix}}_{\dot{\boldsymbol{\nu}}} = \begin{cases}\dfrac{1}{X_{\dot{u}}}\cdot\left(F_u - X_u\cdot u\cdot|u|\right)\\ \dfrac{1}{Y_{\dot{v}}}\cdot\left(F_v - Y_v\cdot v\cdot|v| - Y_{ur}\cdot u\cdot r\right)\\ \dfrac{1}{N_{\dot{u}}}\cdot\left(\Gamma_r - N_r\cdot r\cdot|r|\right)\end{cases}$$

$$\rightarrow \dot{\boldsymbol{\nu}} = \mathbf{f}_D(\boldsymbol{\Theta}, \boldsymbol{\nu}, \boldsymbol{\eta}, \mathbf{F}_B)$$

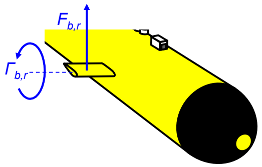
$$\underbrace{\begin{bmatrix}F_u\\ F_v\\ \Gamma_r\end{bmatrix}}_{\mathbf{F}_B} = \underbrace{\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & d\end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix}F_p\\ \left.\begin{matrix}F_{rud}^{Dg}\\ F_{rud}^{Lft}\end{matrix}\right\} = \mathbf{f}_{rud}(v_t, \delta_{rud})\end{bmatrix}^T}_{\mathbf{F}_m}$$

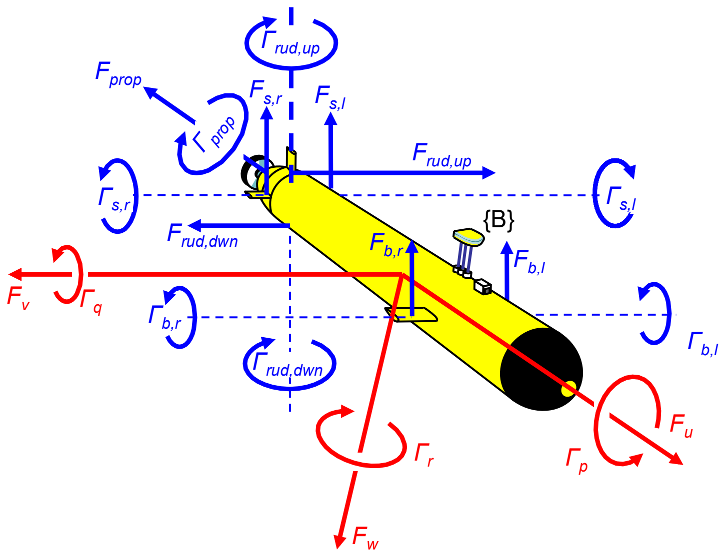
$$\rightarrow \mathbf{F}_B = \mathbf{A}\cdot\mathbf{F}_m$$

Actuation Model

- 3D, Cartesian, Propulsive actuation, AUV

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$





$$\underbrace{\begin{bmatrix} F_{b,l} \\ \Gamma_{b,l} \\ F_{b,r} \\ \Gamma_{b,r} \\ F_{s,l} \\ \Gamma_{s,l} \\ F_{s,r} \\ \Gamma_{s,r} \\ F_{rud,up} \\ \Gamma_{rud,up} \\ F_{rud,dwn} \\ \Gamma_{rud,dwn} \\ F_{prop} \\ \Gamma_{prop} \end{bmatrix}}_{\mathbf{F_m}}$$

$$\mathbf{F_B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R & 0 & -R & 0 & R & 0 & -R & 0 & R & 0 & R & 0 & 0 & 1 \\ d_s & 1 & -d_s & -1 & d_b & 1 & -d_b & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -d_b & 1 & d_b & -1 & 0 & 0 \end{bmatrix} \cdot \mathbf{F_m}$$

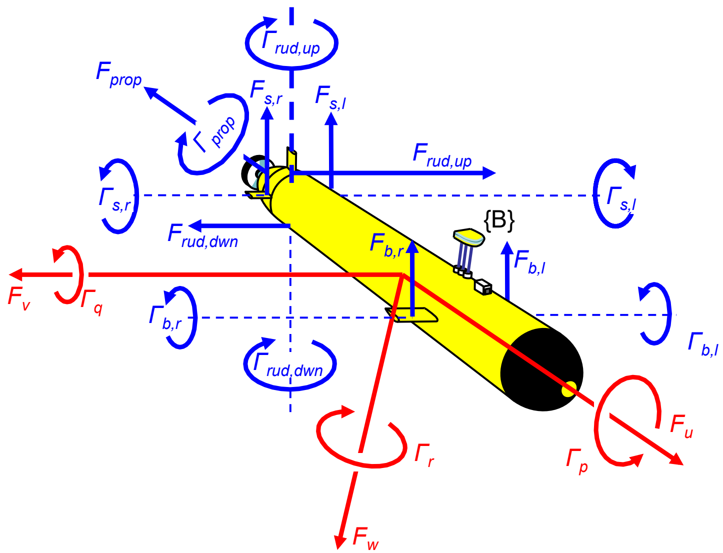
(please check !)

$$\mathbf{F_B} = \mathbf{A} \cdot \mathbf{F_m}$$

Actuation Model

- 3D, Cartesian, Propulsive actuation, AUV

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F_B}}$$

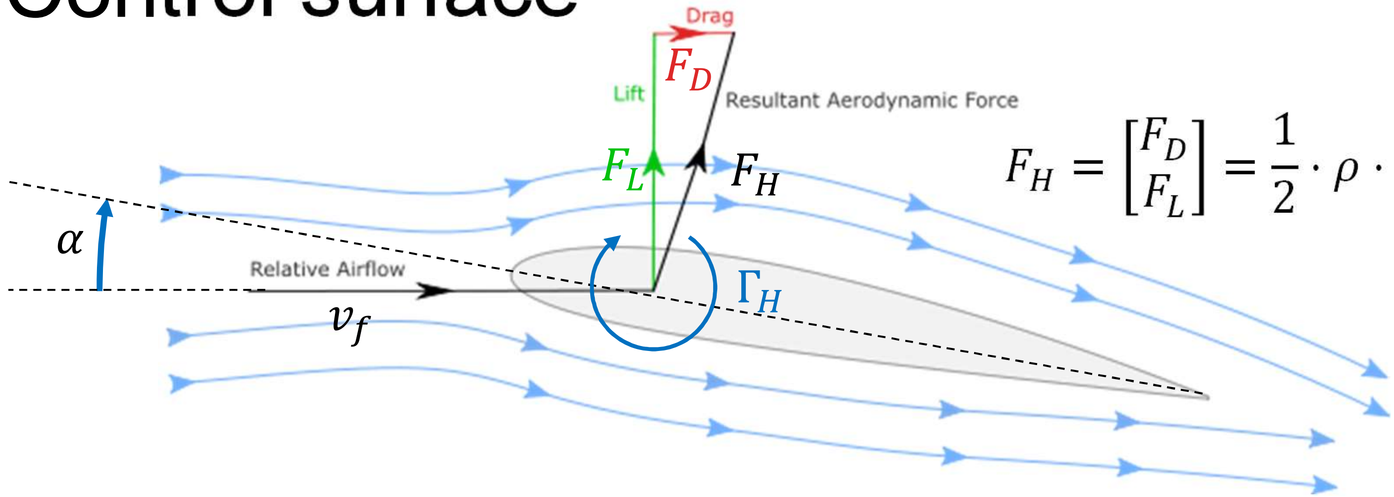


$$\mathbf{F_B} = \mathbf{A} \cdot \mathbf{F_m}$$

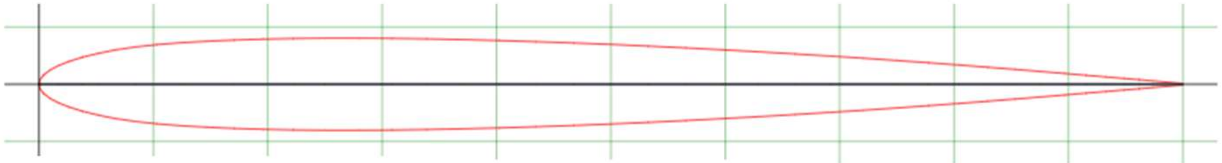
$$\underbrace{\begin{bmatrix} F_{b,l} \\ \Gamma_{b,l} \\ F_{b,r} \\ \Gamma_{b,r} \\ F_{s,l} \\ \Gamma_{s,l} \\ F_{s,r} \\ \Gamma_{s,r} \\ F_{rud,up} \\ \Gamma_{rud,up} \\ F_{rud,dwn} \\ \Gamma_{rud,dwn} \\ F_{prop} \\ \Gamma_{prop} \end{bmatrix}}_{\mathbf{F_m}} \longleftrightarrow \text{Actuators characteristics} \longleftrightarrow \begin{bmatrix} \delta_{rud,up} \\ \delta_{rud,dwn} \\ \delta_{s,r} \\ \delta_{s,l} \\ \delta_{b,r} \\ \delta_{b,l} \\ \omega_{prop} \end{bmatrix}$$

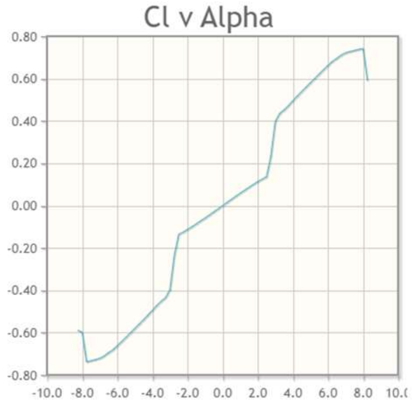
Actuator characteristics

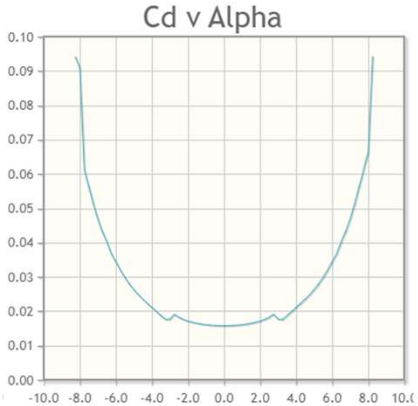
- Control surface

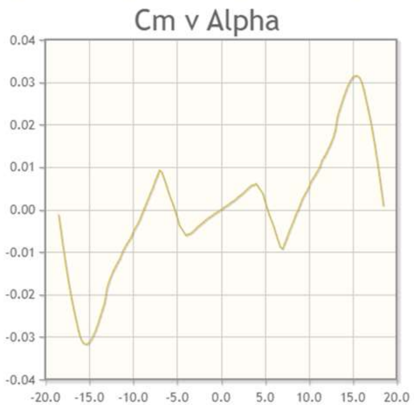


$$F_H = \begin{bmatrix} F_D \\ F_L \end{bmatrix} = \frac{1}{2} \cdot \rho \cdot S \cdot v_f^2 \cdot \begin{bmatrix} C_D(\alpha) \\ C_L(\alpha) \end{bmatrix}$$





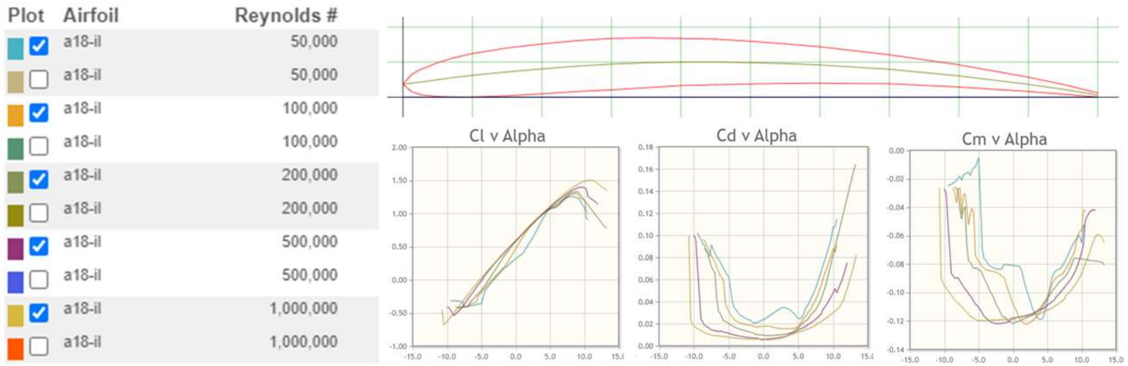


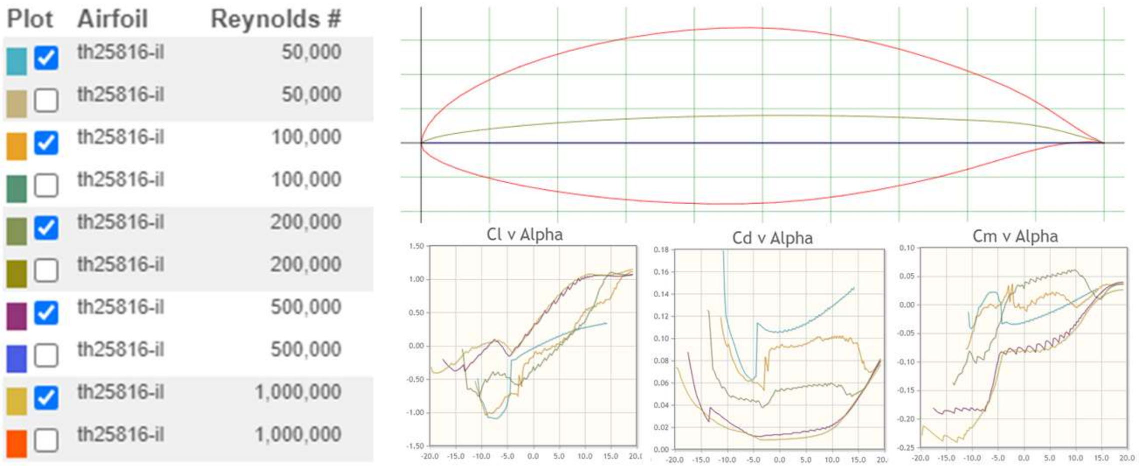


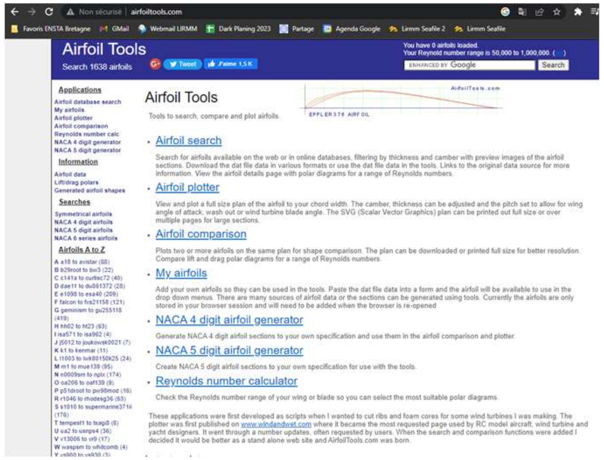
$$\begin{bmatrix} F_D \\ F_L \\ \Gamma_H \end{bmatrix} = \mathbf{f}_{\mathrm{m}}\big(\delta(c_{\mathrm{m}})\big)$$

Actuator characteristics

- Control surface

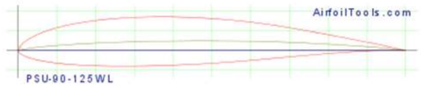




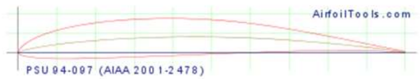


http://airfoiltools.com/airfoil/naca4digit

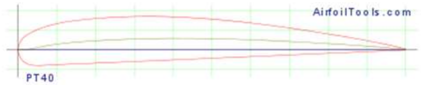
(psu-90-125wl-il) PSU-90-125WL



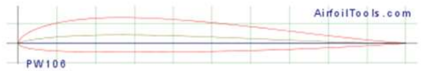
(psu94097-il) PSU 94-097 (AIAA 2001-2478)



(pt40-il) PT40



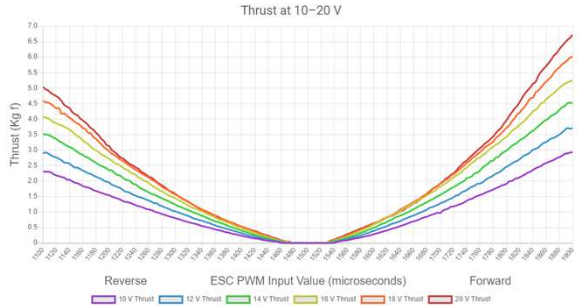
(pw106-pw) PW106



$$\mathbf{F}_{\mathrm{m}} = [F_D, F_L, \Gamma_M]^{\mathrm{T}} = \mathbf{f}_{\mathrm{m}}(C_L(\alpha), C_D(\alpha), C_M(\alpha), v, \delta(c_{\mathrm{m}}), S, \rho)$$

Actuator characteristics

- Thruster (T200, BlueRobotics)



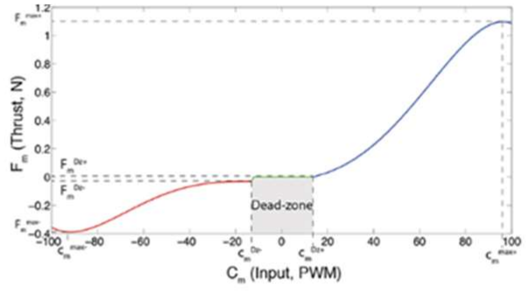
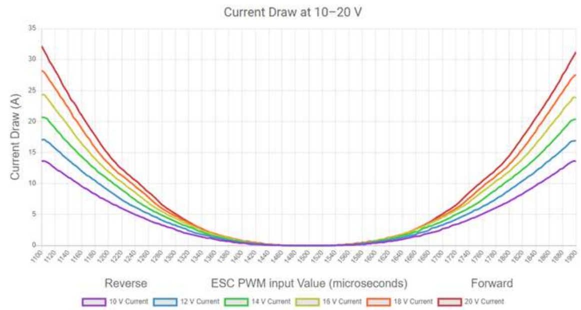
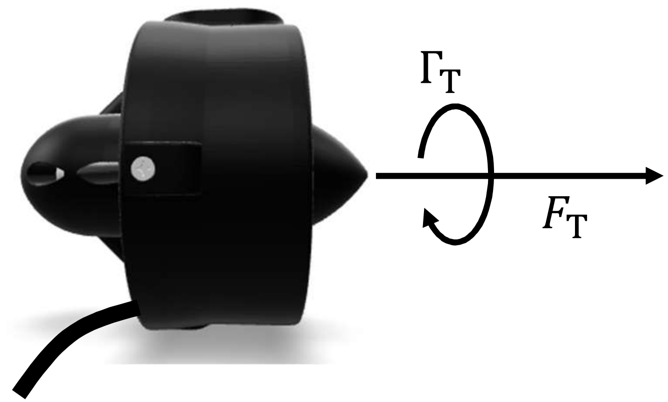


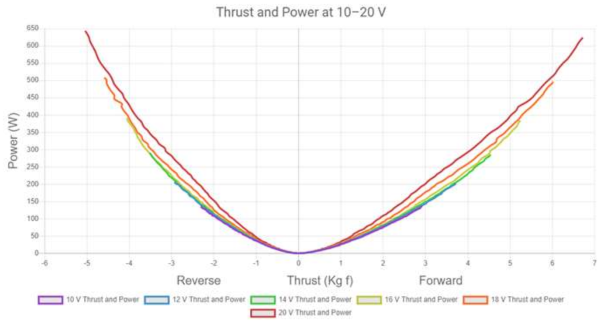
Fig. 4. Motor characteristic identification.





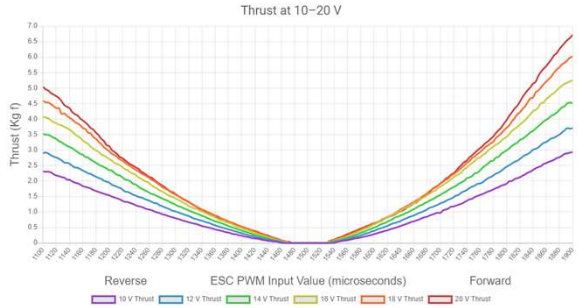
$c_{\mathrm{m}}(\mathrm{PWM})$

$$\begin{bmatrix} F_{\mathrm{T}} \\ \Gamma_{\mathrm{T}} \end{bmatrix} = \mathbf{f}_{\mathrm{m}}(c_{\mathrm{m}})$$



Actuator characteristics

- Thruster (T200, BlueRobotics)



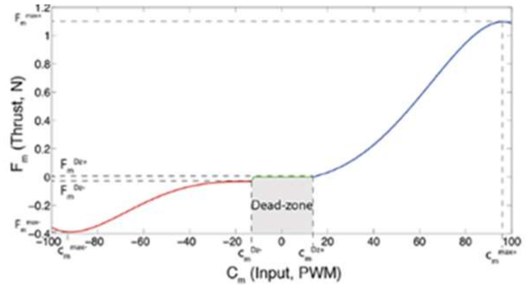
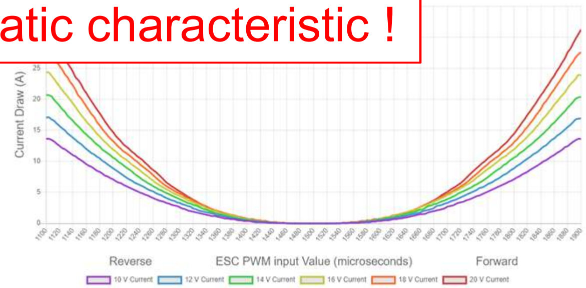
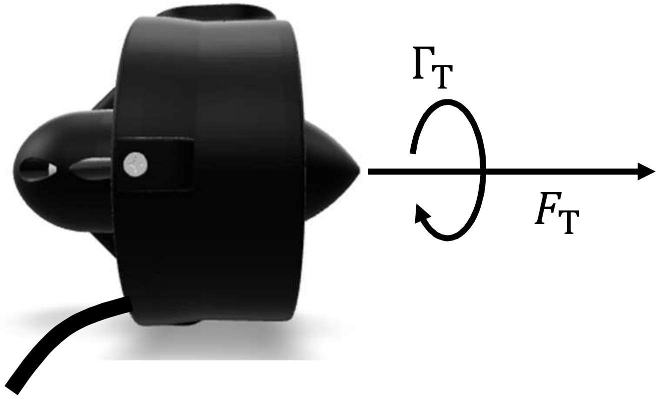


Fig. 4. Motor characteristic identification.

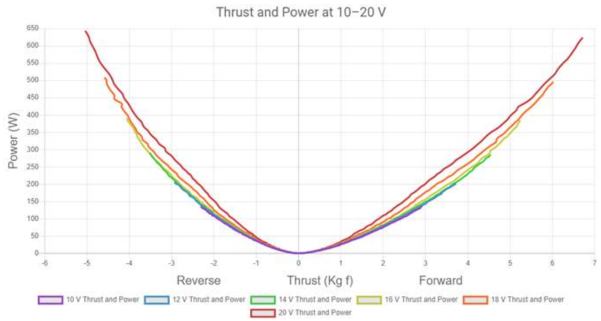
! Static characteristic !





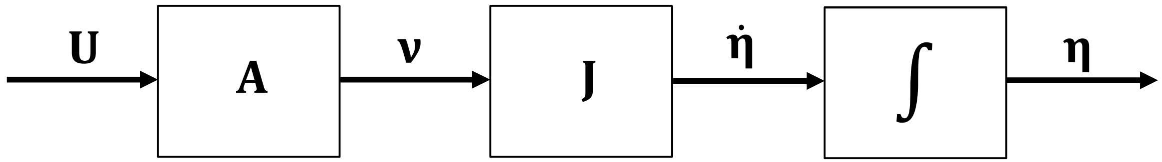
$c_\mathrm{m}(\mathrm{PWM})$

$$\begin{bmatrix} F_\mathrm{T} \\ \Gamma_\mathrm{T} \end{bmatrix} = \mathbf{f}_\mathrm{m}(c_\mathrm{m})$$

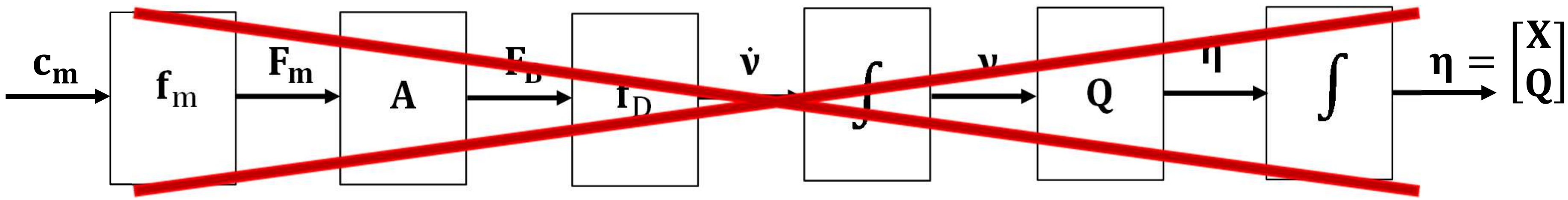


Simulator

- Kinematics

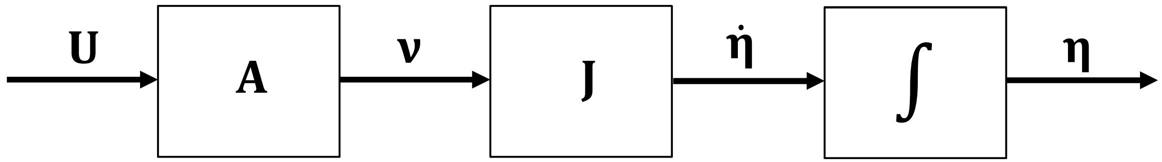


- Dynamics



Simulation

- Kinematics



- Dynamics