# III - Simulation<br>III - Simulation

#### **Representation Formalism**



#### Simulator

• Kinematics



• Dynamics



#### Simulator

• Kinematics<br>• Kinematics





Respect de la contrainte de normalisation ? Utilisation des Contraintes de Lagrange

## Lagrange Constraint Lagrange Constraint<br>
Let the following dynamical system with model :  $F_1 = M \cdot \ddot{x} + G(x, \dot{x})$ , where  $x = [x_1, ..., x_n]^T$ <br>
Considering  $F = F_1 - G(x, \dot{x})$ , yields :<br>  $F = M \cdot \ddot{x}$ **Lagrange Constraint**<br>
Let the following dynamical system with model :  $F_1 = M \cdot \dot{x} + G(x, \dot{x})$ , where  $x = [x_1, ..., x_n]^T$ <br>
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This system undergoes the following constraints:<br>  $\phi$

Lagrange Constantial  $L$ et the following dynamical system with model :  $F_1 = M \cdot \hat{x} + G(x, \hat{x})$ , where  $x = [x_1, ..., x_n]^T$  denotes its state vector.<br>Considering  $F = F_1 - G(x, \hat{x})$ , yields :<br> $F = M \cdot \hat{x}$ Let the following dynamical system with model :  $F_1 = M \cdot \ddot{x} + G(x, \dot{x})$ , where  $x = [x_1, ..., x_n]^T$  denotes its state vector. denotes its state vector. **Lagrange Constraint**<br>
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Considering  $F = F_1 - G(x, \dot{x})$ , yields :<br>  $F = M \cdot \ddot{x}$ <br>
This system undergoes the following co  $\sum$  Constraint<br>  $M \cdot \ddot{x} + G(x, \dot{x})$ , where  $x = [x_1, ..., x_n]^T$  denotes its state vector.<br>  $F = M \cdot \ddot{x}$ <br>  $\phi(x) = 0$ <br>  $\theta \phi(x) |_{x = A, x^* = 0}$  $F_1 = M \cdot \ddot{x} + G(x, \dot{x})$ , where  $x = [x_1, ... x_n]^T$  denotes its sta<br>  $F = M \cdot \ddot{x}$ <br>  $\phi(x) = 0$ <br>  $\phi(x) = 0$ <br>  $\phi(x) = \dot{x} = A \cdot \dot{x} = 0$ <br>  $\frac{\dot{A}}{A} \cdot \dot{x} + A \cdot \ddot{x} = 0$ <br>
Subere  $A^T \cdot \dot{A}$  designs the forces of respect of the contracted  $\bullet$  **COTTS LT CITTLE**<br>  $+ G(x, \dot{x})$ , where  $x = [x_1, ... x_n]^T$  denotes its state vector.<br>  $\cdot \ddot{x}$ <br>  $\cdot \dot{x} = A \cdot \dot{x} = 0$ <br>  $\ddot{x} = 0$ <br>  $\ddot{x} = 0$  $\mathbf{r}_1 = \mathbf{M} \cdot \dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}, \dot{\mathbf{x}})$ , where  $\mathbf{x} = [x_1, ..., x_n]^T$  denotes its state vector.<br>  $\mathbf{F} = \mathbf{M} \cdot \ddot{\mathbf{x}}$ <br>  $\phi(\mathbf{x}) = 0$ <br>  $\cdots$ ,  $\frac{\partial \phi(\mathbf{x})}{\partial x_n} \cdot \dot{\mathbf{x}} = \mathbf{A} \cdot \dot{\mathbf{x}} = 0$ <br>  $\mathbf{\dot{A}} \cdot \dot{\mathbf{x}} + \mathbf{A}$ 

$$
\mathbf{F} = \mathbf{M} \cdot \ddot{\mathbf{x}}
$$

Considering 
$$
\mathbf{F} = \mathbf{F}_1 - \mathbf{G}(\mathbf{x}, \dot{\mathbf{x}})
$$
, yields :  
\n  
\nThis system undergoes the following constraints:  
\n
$$
\phi(\mathbf{x}) = 0
$$
\nThe constraint's derivation provides :  
\n
$$
\frac{\left[\frac{\partial \phi(\mathbf{x})}{\partial x_1}, ..., \frac{\partial \phi(\mathbf{x})}{\partial x_n}\right] \cdot \dot{\mathbf{x}} = \mathbf{A} \cdot \dot{\mathbf{x}} = 0}{\dot{\mathbf{A}} \cdot \dot{\mathbf{x}} + \mathbf{A} \cdot \ddot{\mathbf{x}} = 0}
$$
\nHence the dynamics of the constrained system is, where  $\mathbf{A}^T \cdot \lambda$  designs the forces of respect to  
\n
$$
\mathbf{F} + \mathbf{A}^T \cdot \lambda = \mathbf{M} \cdot \ddot{\mathbf{x}}
$$

 $\texttt{r}\cdot \boldsymbol{\lambda}$  designs the forces of respect of the contraint:

$$
\mathbf{F} + \mathbf{A}^{\mathrm{T}} \cdot \lambda = \mathbf{M} \cdot \ddot{\mathbf{x}}
$$

This system undergoes the following constraints:  
\n
$$
\phi(x) = 0
$$
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\n
$$
\frac{\left[\frac{\partial \phi(x)}{\partial x_1},...,\frac{\partial \phi(x)}{\partial x_n}\right] \cdot \dot{x} = A \cdot \dot{x} = 0}{\dot{A} \cdot \dot{x} + A \cdot \ddot{x} = 0}
$$
\nHence the dynamics of the constrained system is, where  $A^T \cdot \lambda$  designs the forces of respect of the contraint:  
\n
$$
F + A^T \cdot \lambda = M \cdot \ddot{x}
$$
\nThe new constrained system is then written as:  
\n
$$
\begin{cases}\nF = M \cdot \ddot{x} - A^T \cdot \lambda \rightarrow \begin{bmatrix} \ddot{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} M & -A^T \\ A & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} F \\ -\dot{A} \cdot \dot{x} \end{bmatrix} \rightarrow \begin{bmatrix} \ddot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & M & -A^T \\ 0 & A & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \dot{x} \\ F \\ -\dot{A} \cdot \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & M & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \dot{x} \\ F \\ 0 \end{bmatrix}
$$

#### Lagrange Constraint

Exemple : an object, that is assimilable to a ponctual mass m, is falling in the vertical plane of a terrestrial gravity field of magnitude  $g$ , with a linear viscous friction with coefficient  $f$ . Its initial position is **Lagravity** field of magnitude g, with a linear viscous friction with coefficient f. Its initial position is denoted  $\mathbf{x}(0) = [0,0]^T$ . This object is attached to a nonelastic rope of length l attached to the point  $\mathbf{x}_R$ **. Laying the computer of a nonelastic rope of length** *i* **attached to a nonelastic rope of length** *i* **attached to the point**  $\mathbf{x}_R = [l/2,0]^T$ **.<br>This object is attached to a nonelastic rope of length** *l* **attached to the poi i** a ponctual mass m, is falling in the vertical plane of a terrestrial viscous friction with coefficient *f*. Its initial position is denoted in nonelastic rope of length *l* attached to the point  $X_R = [l/2, 0]^T$ .<br>  $-\frac{$ **train the vertical plane of a terrestrial**<br>on the vertical plane of a terrestrial<br>of f. Its initial position is denoted<br>tached to the point  $X_R = [l/2,0]^T$ . **Graph**<br> **Solution Control and Solution Section**<br> **Solution**<br> **Solution**<br> **Solution**<br> **Solution**<br> **Solution**<br> **Solution**<br> **F** = **M** · **X** =  $\begin{bmatrix} 0 \\ -m \end{bmatrix}$  - **f** · **X**, where **M** =  $\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$ <br>  $\rightarrow \ddot{x} = M^{-$ **Constraint**<br>
mass *m*, is falling in the vertical plane of a terrestrial<br>
ion with coefficient *f*. Its initial position is denoted<br>
rope of length *l* attached to the point  $X_R = [l/2,0]^T$ .<br>  $\cdot \dot{x}$ , where  $M = \begin{bmatrix} m & 0 \\$ 

**Lagrange Constrained dynamics**\n
$$
\begin{aligned}\n\text{Example: an object, that is assimilable to a potential mass } m, \text{ is falling in the vertical plane of a terrestrial gravity field of magnitude } g, \text{ with a linear viscous friction with coefficient } f. \text{ Its initial position is denoted } \mathbf{X}(0) = [0,0]^T. \text{ This object is attached to a nonelastic rope of length } l \text{ attached to the point } \mathbf{X}_R = [l/2,0]^T.\n\end{aligned}
$$
\nUnconstrained dynamics:  $\mathbf{F} = \mathbf{M} \cdot \ddot{\mathbf{x}} = \begin{bmatrix} 0 & g \ -m \cdot g \end{bmatrix} - f \cdot \dot{\mathbf{x}}, \text{ where } \mathbf{M} = \begin{bmatrix} m & 0 \ 0 & m \end{bmatrix}$ 

\n
$$
\rightarrow \ddot{\mathbf{x}} = \mathbf{M}^{-1} \cdot \mathbf{F} \rightarrow \begin{bmatrix} \ddot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_d \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{F} \\ \dot{\mathbf{x}} \end{bmatrix}
$$
\nConstrained dynamics:  $\mathbf{M} \times \mathbf{S} = \mathbf{M}^{-1} \cdot \mathbf{F} \rightarrow \begin{bmatrix} \ddot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_d \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{F} \\ \dot{\mathbf{x}} \end{bmatrix}$ 

**Example:** an object, that is assimilable to a particular mass *m*, is falling in the vertical plane of a terrestrial  
gravity field of magnitude *g*, with a linear viscosity function with coefficient *f*. Its initial position is denoted  

$$
X(0) = [0,0]^T
$$
. This object is attached to a nonelastic rope of length *l* attached to the point  $X_R = [l/2,0]^T$ .  
Inconstrained dynamics:  $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} = \begin{bmatrix} 0 & 0 \\ -m \cdot g \end{bmatrix} - f \cdot \hat{\mathbf{x}}$ , where  $\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}$   
Constrained dynamics:  $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} = \begin{bmatrix} -m \cdot g \end{bmatrix} - f \cdot \hat{\mathbf{x}}$ , where  $\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{F} \\ \mathbf{X} \end{bmatrix}$   
Constrained dynamics:  $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} = \mathbf{A} \cdot \hat{\mathbf{x}} + \mathbf{A} \cdot \hat{\mathbf{x}} = 2 \cdot \hat{\mathbf{x}} \cdot \hat{\mathbf{x}}^{-1}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;  
 $\mathbf{F} = \mathbf{M} \cdot \hat{\mathbf{x}} - \mathbf{A}^{-1} \cdot \hat{\mathbf{x}}$ ;<

If 
$$
\phi(\mathbf{X}) < 0
$$
, unconstrained dynamics :  $\begin{bmatrix} \ddot{\mathbf{X}} \\ \dot{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_d \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{F} \\ \dot{\mathbf{X}} \end{bmatrix}$   
else, constrained dynamics : (at first instant :  $\dot{\mathbf{X}}(2) = 0$ ) and  $\begin{bmatrix} \ddot{\mathbf{X}} \\ \dot{\mathbf{X}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0} & -\mathbf{A}^T \\ \mathbf{0} & \mathbf{I}_d & \mathbf{0} \\ \mathbf{A} & \dot{\mathbf{A}} & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{F} \\ \dot{\mathbf{X}} \\ 0 \end{bmatrix}$ 

#### Lagrange Constraint

**Lagrange Constraint**<br>Application to Quaternion integration (1/2):<br>The position and attitude of a mobile system are denoted :  $\eta = \begin{bmatrix} x \\ 0 \end{bmatrix}$ , while its body-frame (a<br>are :  $\mathbf{v} = \begin{bmatrix} v_B = [u, v, w]^T \end{bmatrix}$ . The kine **Lagrange Constraint**<br>Application to Quaternion integration (1/2):<br>The position and attitude of a mobile system are denoted :  $\eta = \begin{bmatrix} X \\ 0 \end{bmatrix}$ , while its body-frame (absolute) velocities<br>are :  $v = \begin{bmatrix} v_B = [u, v, w]^T \ w_B$  $X$  while its hody from  $($ ohoolute  $\mathbf{Q}$ , which is body-name (absolute) ve  $\begin{split} \text{M} & \text{S} & \text{M} \end{split}$ <br>, while its body-frame (absolute) velocities<br>as :<br> $\begin{split} \text{M} & \times \text{M}^* \end{split}$ are :  $\mathbf{v} = \begin{bmatrix} \mathbf{V}_{\mathrm{B}} = [u, v, w]^{\mathrm{T}} \\ \mathbf{W} \end{bmatrix}$ . The kinematic model can be expr  $W_{\text{B}} = [p, q, r]^{T}$ . The kinematic moder can be ex**and the sum of the control of the control of the control of a mobile system are denoted :**  $\eta = \begin{bmatrix} x \\ 0 \end{bmatrix}$ **, while its body-frame (absolute) velocities<br>
The kinematic model can be expressed as :<br> \eta = \begin{bmatrix} x \\ 0 \end{bmatrix}**  $\dot{\eta} = \begin{vmatrix} \dot{\alpha} \\ \dot{\alpha} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & \frac{T}{T} \end{vmatrix}$  $\mathbf{\dot{X}} = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  $\mathbf{T}_{\mathrm{Q}}^{\mathrm{v}}\cdot\left(\mathbf{Q}\otimes\left[0,\mathbf{V}_{\boldsymbol{B}}^{T}\right]^{T}\otimes\mathbf{Q}^{*}\right)\right]$  $1$   $2.2$   $10$   $7.1$   $1$  $\frac{1}{2} \cdot \mathbf{Q} \otimes \underbrace{[0, \boldsymbol{\omega}_B^T]^T}_{\mathbf{Q}}$  $\Omega_B$  , and the set of  $\Box$ where  $\mathbf{T}_{\mathrm{Q}}^{\mathrm{v}}=\begin{vmatrix} 0 & 0 & 1 & 0 \end{vmatrix}$ , which allows to transfo **Lagrange Constraint (11)**<br>
Dependent on the system are denoted :  $\eta = \begin{bmatrix} x \\ 0 \end{bmatrix}$ , while its if<br>  $\eta = [u, v, w]^T$ <br>  $\eta = [p, q, r]^T$ . The kinematic model can be expressed as :<br>  $\dot{\eta} = \begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} \tau_0^v \cdot (\mathbf{Q} \$ **Lagrange Constraint (12):**<br>
and attitude of a mobile system are denoted :  $\eta = \begin{bmatrix} x \\ 0 \end{bmatrix}$ , while its  $\theta = [u, v, w]^T$ <br>  $= [v, q, r]^T$ . The kinematic model can be expressed as :<br>  $\dot{\eta} = \begin{bmatrix} \dot{x} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} \tau_0^x$ **CONSTRANGE CONSTRANGE (12):**<br>
and attitude of a mobile system are denoted :  $\eta = \begin{bmatrix} x \\ 0 \end{bmatrix}$ , while its l<br>  $= [u, v, w]^T$ . The kinematic model can be expressed as :<br>  $\eta = \begin{bmatrix} x \\ \dot{\mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_0^{\gamma} \cdot (\mathbf{Q} \$ **Solution (1/2):**<br>
Integration (1/2):<br>
of a mobile system are denoted :  $\eta = \begin{bmatrix} X \\ 0 \end{bmatrix}$ , while its body-frame (absolute) velocities<br>
The kinematic model can be expressed as :<br>  $\eta = \begin{bmatrix} X \\ 0 \end{bmatrix} = \begin{bmatrix} T_0^y \cdot (Q \otimes$ The inverse transformations uses  $\texttt T_{\textsc{v}}^{\texttt{Q}} = \left(\texttt T_{\texttt{Q}}^{\texttt{v}}\right)^{\text{T}}$ . Hence the inverse kinematic mode **. Hence the inverse kinematic model is written as :**<br>  $\begin{bmatrix}\n\mathbf{T}_Q^V \cdot \left( \mathbf{Q} \otimes [0, \mathbf{V}_B^T]^T \otimes \mathbf{Q}^* \right)\n\end{bmatrix}$ <br> **Example 18 and be expressed as :**<br>  $\begin{bmatrix}\n\mathbf{T}_Q^V \cdot \left( \mathbf{Q} \otimes [0, \mathbf{V}_B^T]^T \otimes \mathbf{Q}^* \right)\n\end{$  $\alpha : \eta = [Q]$ , while its body-frame (absolute) velocities<br>
expressed as :<br>  $\otimes [0, v_B^T]^T \otimes Q^*]$ <br>  $Q \otimes [0, \omega_B^T]^T$ <br>
oure imaginary quaternion into its equivalent vector.<br>
ie inverse kinematic model is written as :<br>  $*\otimes [0, X$ are :  $\mathbf{v} = [\mathbf{w}_B - [p, q, r]^T]$ . The kinematic model can be expressed as :<br>  $\mathbf{\dot{n}} = [\mathbf{\dot{\tilde{Q}}}] = \begin{bmatrix} \mathbf{T}_{\mathbf{Q}}^{\times} \cdot (\mathbf{Q} \otimes [0, \mathbf{v}_B^T]^T \otimes \mathbf{Q}^*) \\ \frac{1}{2} \cdot \mathbf{Q} \otimes \frac{[\mathbf{O}, \mathbf{\omega}_B^T]^T}{\mathbf{\Omega}_B} \end{bmatrix}$ <br>
where  $\$  $\vec{\eta} = \begin{bmatrix} \vec{x} \\ \vec{Q} \end{bmatrix} = \begin{bmatrix} \vec{\eta}_d \cdot (\vec{Q} \otimes [0, \omega_B^T]^T) \\ \frac{1}{2} \cdot \vec{Q} \otimes [\underline{0}, \omega_B^T]^T \end{bmatrix}$ <br>  $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , which allows to transform a pure imaginary quaternion into its equiv

$$
\mathbf{v} = \begin{bmatrix} \mathbf{V}_{\mathrm{B}} \\ \mathbf{W}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{\mathrm{v}}^{\mathrm{Q}} \cdot \left( \mathbf{Q}^{*} \otimes \begin{bmatrix} 0, \dot{\mathbf{X}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \otimes \mathbf{Q} \right) \\ \mathbf{T}_{\mathrm{v}}^{\mathrm{Q}} \cdot \left( 2 \cdot \mathbf{Q}^{*} \otimes \dot{\mathbf{Q}} \right) \end{bmatrix}
$$

The inverse dynamic model of the system is expressed in the body frame as  $: \begin{bmatrix} a_B \\ v_B \end{bmatrix} = M^{-1} \cdot (F_B(c_m) - f(v, \eta)),$ The inverse dynamic model of the system is expressed in the body frame as  $\begin{bmatrix} \begin{bmatrix} \begin{matrix} \mathbf{B} \end{bmatrix} = \mathbf{M}^{-1} \cdot (\mathbf{F}_{\mathbf{B}}(\mathbf{c}_{\mathbf{m}}) - \mathbf{f}(\mathbf{v},\mathbf{\eta})) \end{bmatrix} \end{bmatrix}$ ,<br>where  $\begin{bmatrix} \mathbf{a}_{\mathbf{B}} \\ \mathbf{v}_{\mathbf{c}} \end{bmatrix}$   $\gamma_{\rm B}$ ] deriotes the (absolute) iongituding  $\begin{pmatrix} [a_B] \ \mathbf{v}_B \end{pmatrix} \neq \dot{\mathbf{v}}$ ). Hence, system dynamics ca  $\left[\mathbf{a}_{\mathrm{B}}\right] \neq \mathbf{\dot{v}}$ ). Hence, system dynamics can be written as :  $\mathbf{\ddot{\eta}} = \begin{bmatrix} \ddot{\mathbf{x}} \ \ddot{\mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{\mathrm{Q}}^{\mathrm{v}} \cdot \left(\mathbf{Q} \otimes \left[0, \mathbf{a}_{\mathrm{B}}^{\mathrm{T}}\right]^{T} \otimes \mathbf{Q}^{*}\right) \ \mathbf{Q} \otimes \mathbf{Q} \otimes \mathbf{Q}^{*}$  $\mathbf{H}_{\mathbf{Q}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , which allows to transform a pure imaginary quaternion into its equivalent vectures transformations uses  $\mathbf{T}_{\mathbf{V}}^{\mathbf{Q}} = (\mathbf{T}_{\mathbf{Q}}^{\mathbf{V}})^{\mathbf{T}}$ . Hence  $\mathbf{Q} \otimes \left[ \mathbb{0}, \boldsymbol{\gamma}_{\text{B}}^{\text{T}} \right]^{T} \otimes \mathbf{Q}^*$ 

### Lagrange Constraint **Lagrange Constraint**<br>Application to Quaternion integration (2/2):<br>The constraint to be considered concerns the quaternion normalisation, that can be written **Lagrange Constraint**<br>Application to Quaternion integration (2/2):<br>The constraint to be considered concerns the quaternion normalisation, that can be written as :<br> $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 = 0$ Constraint<br>
ion normalisation, that can be written as :<br>  $\cdot$  Q – 1 = 0<br>
Q<sup>T</sup> **Lagrange Constraint dependent Constraint**<br>Application to Quaternion integration (2/2):<br>The constraint to be considered concerns the quaternion normalisation, that<br> $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 = 0$ <br>First derivation yields **agram and Constraint**<br>
Integration (2/2):<br>
dered concerns the quaternion normalisation, that can be written as<br>  $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 = 0$ <br>  $\cdot \dot{\mathbf{Q}} = \mathbf{A} \cdot \dot{\mathbf{Q}} = 0$ , where  $\mathbf{A} = \mathbf{Q}^T$ <br>  $-\mathbf{A} \cdot \ddot{\math$ **Lagrange Constraint**<br>Application to Quaternion integration (2/2):<br>The constraint to be considered concerns the quaternion normalisation, that can be writte<br> $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 = 0$ <br>First derivation yields :  $\mathbf$

$$
\phi(\mathbf{Q}) = \mathbf{Q}^{\mathrm{T}} \cdot \mathbf{Q} - 1 = 0
$$

First derivation yields :  $\mathbf{Q}^T \cdot \dot{\mathbf{Q}} = \mathbf{A} \cdot \dot{\mathbf{Q}} = 0$ , where  $\mathbf{A} = \mathbf{Q}^T$ T<sub>1</sub>

Application to Quaternion integration (2/2):<br>
The constraint to be considered concerns the quaternion normalisation, that can be written as :<br>  $\phi(Q) = Q^T \cdot Q - 1 = 0$ <br>
First derivation yields :  $Q^T \cdot Q = A \cdot Q = 0$ , where  $A = Q^T$ Application to Quaternion integration (2/2):<br>
The constraint to be considered concerns the quaternion norm<br>  $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 =$ <br>
First derivation yields :  $\mathbf{Q}^T \cdot \mathbf{Q} = \mathbf{A} \cdot \mathbf{Q} = 0$ , where  $\mathbf{A} = \math$ The constraint to be considered concerns the quaternion normalisation, that can be written as :<br>  $\phi(\mathbf{Q}) = \mathbf{Q}^T \cdot \mathbf{Q} - 1 = 0$ <br>
First derivation yields :  $\mathbf{Q}^T \cdot \mathbf{Q} = \mathbf{A} \cdot \mathbf{Q} = 0$ , where  $\mathbf{A} = \mathbf{Q}^T$ <br>

$$
\ddot{\mathbf{\eta}} = \begin{bmatrix} \ddot{\mathbf{X}} \\ \ddot{\mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{\mathrm{Q}}^{\mathrm{v}} \cdot \left( \mathbf{Q} \otimes \begin{bmatrix} 0, \mathbf{a}_{\mathrm{B}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \otimes \mathbf{Q}^* \right) \\ \mathbf{Q} \otimes \begin{bmatrix} 0, \gamma_{\mathrm{B}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \otimes \mathbf{Q}^* \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \lambda \cdot \mathbf{A}^{\mathrm{T}} \end{bmatrix}
$$

$$
\begin{bmatrix} \ddot{\eta} \\ \dot{\eta} \\ \dot{\lambda} \end{bmatrix} = \underbrace{\begin{bmatrix} I_d & \mathbf{0} \\ (\mathbf{0} \times \mathbf{0}) & \begin{bmatrix} \mathbf{0} \\ -\mathbf{A}^T \end{bmatrix} \\ \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ (\mathbf{0} \times \mathbf{0}) & \mathbf{I}_d \\ (\mathbf{0} \times \mathbf{0}) & \begin{bmatrix} \mathbf{0} \\ (\mathbf{0} \times \mathbf{0}) \end{bmatrix} \\ \mathbf{0} \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \cdot \begin
$$

#### Simulator: Integration

Numerical solution of an ODE

 $x^{(n)}(t) = f(x^{(n-1)}(t), ..., \dot{x}(t), x(t), u^{(m)}(t), ..., \dot{u}(t), P) \rightarrow x(t)$ Initial conditions at  $t_0: x^{(n-1)}(t_0), ..., x^{(i)}(t_0), ..., \dot{x}(t_0), x(t_0)$  et  $u^{(m)}(t), ..., \dot{u}(t)$ , P known  $\rightarrow$  Computat<sup>o</sup> of  $x^{(n)}(t_0) = f(x^{(n-1)}(t_0), ..., x(t_0), x(t_0), u^{(m)}(t_0), ..., u(t_0), P)$ 

 $\rightarrow$  Computation at  $(t_0 + dt)$ :  $x^{(n-1)}(t_0 + dt)$ , ...,  $x^{(i)}(t_0 + dt)$ , ...,  $\dot{x}(t_0 + dt)$ ,  $x(t_0 + dt)$ 

Integration over an horizon  $dt$  :  $x^{(n-1)}(t_0 + dt) = x^{(n-1)}(t_0) + \int_{0}^{t_0 + dt} x^{(n)}(\tau) d\tau$ <br>  $x^{(i-1)}(t_0 + dt) = x^{(i-1)}(t_0) + \int_{t_0}^{t_0 + d} x^{(i)}(\tau) d\tau$ <br>  $x(t_0 + dt) = x(t_0) + \int_{t_0}^{t_0 + d} \dot{x}(\tau) d\tau$ 

#### Simulator: Integration

Numerical solution of an ODE

 $x^{(n)}(t) = f(x^{(n-1)}(t), ..., x(t), x(t), u^{(m)}(t), ..., u(t), P) \rightarrow x(t)$ 

Initial conditions at  $t_0: x^{(n-1)}(t_0), ..., x^{(i)}(t_0), ..., \dot{x}(t_0), x(t_0)$  et  $u^{(m)}(t), ..., \dot{u}(t)$ , P known

 $\rightarrow$  Computat<sup>o</sup> of  $x^{(n)}(t_0) = f(x^{(n-1)}(t_0), ..., x(t_0), x(t_0), u^{(m)}(t_0), ..., u(t_0), P)$ 

 $\rightarrow$  Computation at  $(t_0 + dt)$ :  $x^{(n-1)}(t_0 + dt)$ , ...,  $x^{(i)}(t_0 + dt)$ , ...,  $\dot{x}(t_0 + dt)$ ,  $x(t_0 + dt)$ 

Integration over an horizon  $dt$  :  $\begin{bmatrix} x^{(n-1)}(t_0 + dt) \\ x^{(n-2)}(t_0 + dt) \\ \vdots \\ x^{(i-1)}(t_0 + dt) \\ \vdots \\ x(t_0 + dt) \end{bmatrix} = \begin{bmatrix} x^{(n-1)}(t_0) \\ x^{(n-2)}(t_0) \\ \vdots \\ x^{(i-1)}(t_0) \\ \vdots \\ x(t_0) \end{bmatrix} + \int_{t_0}^{t_0 + d} \begin{bmatrix} x^{(n)}(\tau) \\ x^{(n-1)}(\tau) \\ \vdots \\ x^{(i$  $\chi(t_0+dt)=\chi(t_0)+\int_{t_0}^{t_0+dt}\dot{\chi}(\tau)\cdot d\tau$ 

#### Simulator: Integration

Numerical solution of an ODE

 $x^{(n)}(t) = f(x^{(n-1)}(t), ..., \dot{x}(t), x(t), u^{(m)}(t), ..., \dot{u}(t), P) \rightarrow x(t)$ 

Initial conditions at  $t_0$ :  $x^{(n-1)}(t_0)$ , ...,  $x^{(i)}(t_0)$ , ...,  $\dot{x}(t_0)$ ,  $x(t_0)$  et  $u^{(m)}(t)$ , ...,  $\dot{u}(t)$ , P known

- $\rightarrow$  Computat<sup>o</sup> of  $x^{(n)}(t_0) = f(x^{(n-1)}(t_0), ..., \dot{x}(t_0), x(t_0), u^{(m)}(t_0), ..., \dot{u}(t_0), P)$
- $\rightarrow$  Computation at  $(t_0 + dt)$ :  $x^{(n-1)}(t_0 + dt)$ , ...,  $x^{(i)}(t_0 + dt)$ , ...,  $\dot{x}(t_0 + dt)$ ,  $x(t_0 + dt)$



## Simulator : Integration **Simulator : Integration**<br>• Numerical solution of an ODE<br> $x^{(n)}(t) = f(x^{(n-1)}(t), ..., \dot{x}(t), x(t), u^{(m)}(t), ..., \dot{u}(t), P) \rightarrow x(t)$ ?

 $x^{(n)}(t) = f(x^{(n-1)}(t), ..., \dot{x}(t), x(t), u^{(m)}(t), ..., \dot{u}(t), P) \rightarrow x(t)$ ?  $\chi_0: \chi^{(n-1)}(t_0), \ldots, \chi^{(i)}(t_0), \ldots, \dot{\chi}(t_0), \chi^{(i)}$  $e^{i)}(t_0),\ldots,\dot x(t_0),x(t_0)$  et  $u^{(m)}(t),\ldots,\dot u(t)$ , P known  $^{(n)}(t_0) = f(x^{(n-1)}(t_0), \ldots, \dot{x}(t_0), x(t_0), u^{(m)}(t_0), \ldots, \dot{u}(t_0), P)$  $m(t_1)$   $i(t_2)$  P  $(0, 0, \ldots, u(t_0), r)$  $s_0 + dt$ ) :  $x^{(n-1)}(t_0 + dt)$ , ...,  $x^{(i)}(t_0 + dt)$ ,  $^{(i)}(t, + dt)$  $_0 + u_1, ..., x_{\lfloor t_0 \rfloor} + u_1, x_{\lfloor t_0 \rfloor} + u_1)$  $(n)(t_0 + dt) = f(x^{(n-1)}(t_0 + dt), \dots, \dot{x}(t_{0+dt}), x(t_0 + dt) \dots)$  $t_0 + 2dt$ ) :  $x^{(n-1)}(t_0 + 2dt)$ , ...,  $x^{(i)}(t_0 + 2dt)$  $^{(i)}(t_0 + 2dt)$  $_0 +$  2al), ...,  $x(t_0 + 2at)$ ,  $x(t_0 + 2at)$  $\rightarrow \cdots$ 

 $s_0 + kdt$ ) :  $x^{(n-1)}(t_0 + kdt)$ , ...,  $x^{(i)}(t_0 + kdt)$  $i(t_{\rm t} + k dt)$  $_0$  +  $\kappa$ ul), ...,  $x(t_0 + \kappa u t)$ ,  $x(t_0 + \kappa u t)$ 

$$
x(t) = \{x(t_0), x(t_0 + dt), ..., x(t_0 + kdt), ..., x(t_0 + ndt)\}
$$
  
Trajectory

#### Simulator : structure



#### Simulator: structure





#### Simulator: structure



#### Simulator : structure



**Physics compuation<br>Physics computation<br>Display Display**