III – Redundancy Management

Redundancy

- Redundant actuation system
  - System that has more actuators than degrees of freedom
  - A system can have a redundant actuation system, while remaining under-actuated
    - Only subsets of Dofs can be redundant
  - Opportunities in the control allocation problem:

- Exemple de systemes redondants
- Jack, Ulysse, Taipan, BlueRov

Redundancy

- The Control Allocation problem :
  - Given the actuation system :

$$\mathbf{F}_{\mathrm{B}} = \underbrace{\mathbf{A}}_{\text{‘Concentrator’}} \cdot \mathbf{F}_{\mathrm{m}}$$

  - Compute the desired actuators’ force, $\mathbf{F}_{\mathrm{m}}^{\mathrm{d}}$, in order to produce a prescribed resulting action $\mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$ on the system.

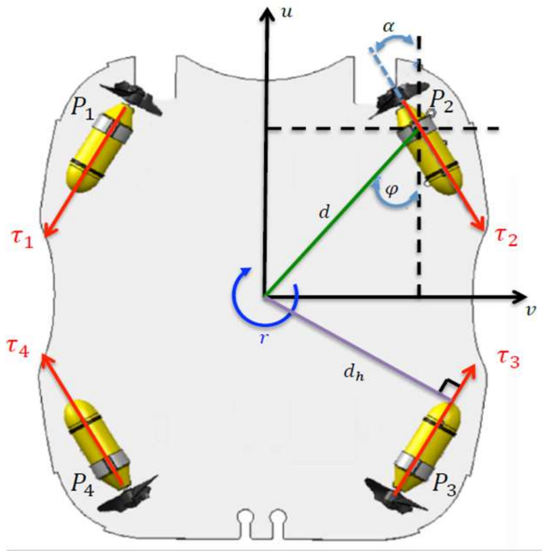
$$\mathbf{F}_{\mathrm{m}}^{\mathrm{d}} = \underbrace{\mathbf{D}}_{\text{‘Dispatcher’}} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

    - If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} > n$, under-determined
    - If $\det(\mathbf{A}) \neq 0$, $\mathbf{D}$ is unique and $\mathbf{D} = \mathbf{A}^{-1}$
    - If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} < n$, $\mathbf{D}$ is not unique

Redundancy

- Example : Jack
  - 6 actuators for 6 Dof
  - redundant in the H plane
  - Globally underactuated

Video JACK



$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix}}_{\mathbf{F}_\mathrm{B}} = \underbrace{\begin{bmatrix} -\cos\varphi & -\cos\varphi & \cos\varphi & \cos\varphi \\ -\sin\varphi & \sin\varphi & \sin\varphi & -\sin\varphi \\ d & -d & d & -d \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{\mathbf{F}_\mathrm{m}}$$

$$\mathbf{F}_\mathrm{B} = \mathbf{A} \cdot \mathbf{F}_\mathrm{m}$$

Dead zone

Redundancy

- Example : Jack
  - Use of the Moore-Penrose pseudo inverse

$$\mathbf{A}^{+} = \mathbf{A}^{\mathrm{T}} \cdot \left(\mathbf{A} \cdot \mathbf{A}^{\mathrm{T}}\right)^{-1} \qquad \mathbf{F}_{\mathrm{m}} = \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

  - Test on Jack, yaw regulation

Presentation manip

$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$

Dead zone

Redundancy

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  - Use of the Moore-Penrose pseudo inverse

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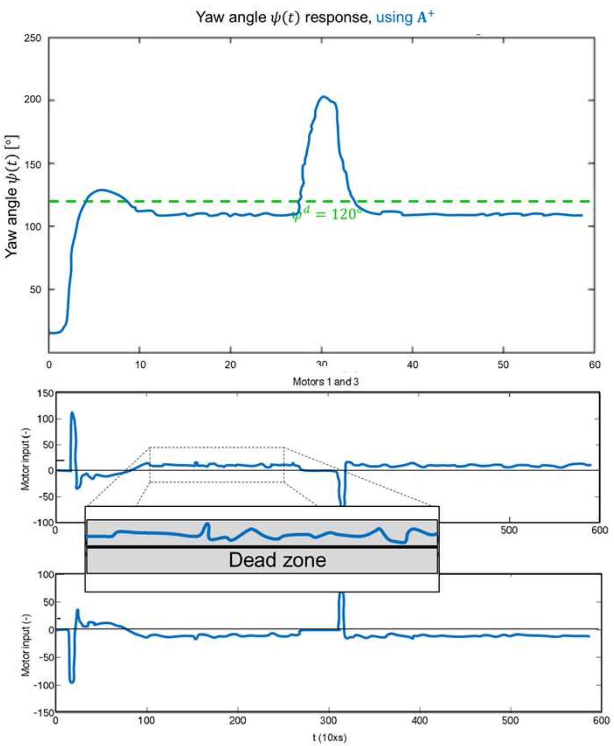
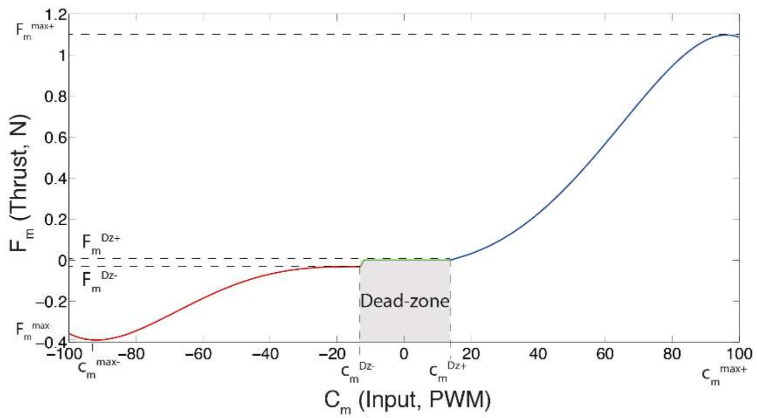
  - Test on Jack, yaw regulation



$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$



Redundancy

- Example : Jack
  - Use of the Moore-Penrose pseudo inverse

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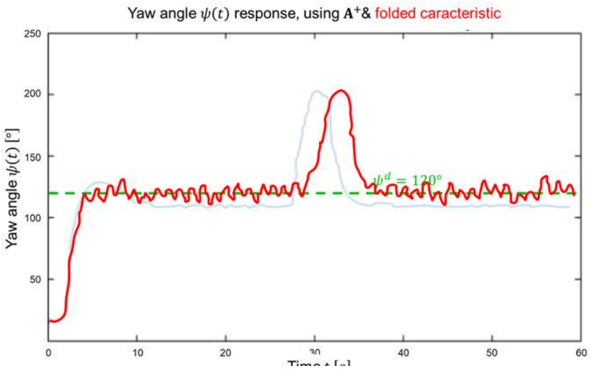
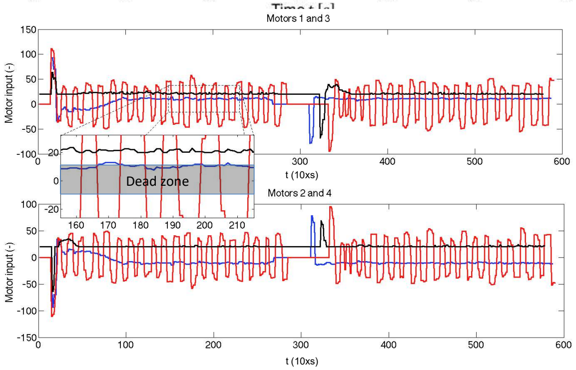
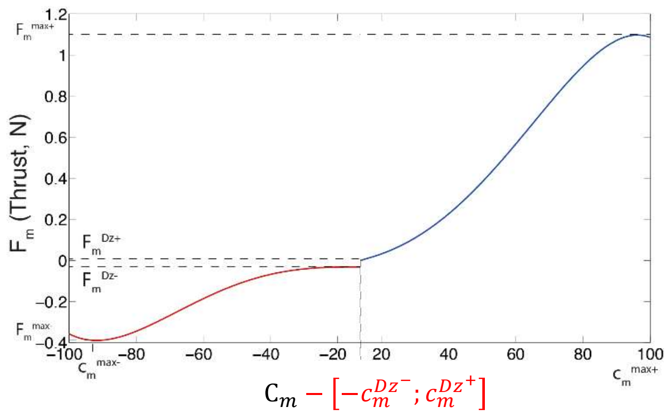
  - Test on Jack, yaw regulation, folding the DZ





$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$

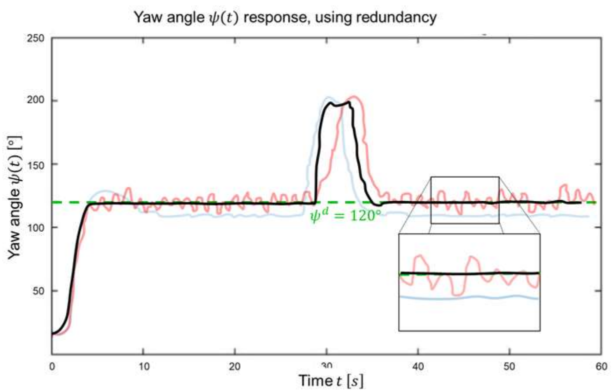


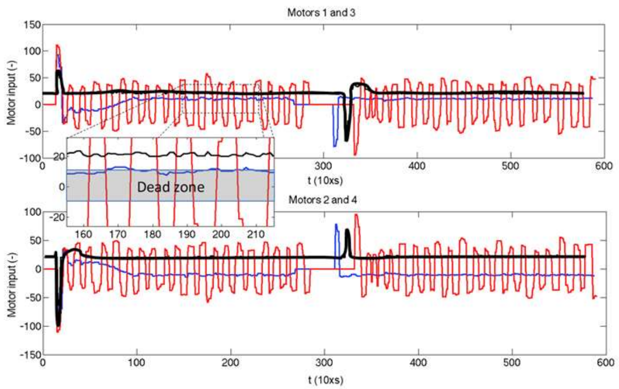
Dead zone

Redundancy

- Example : Jack
  - Use redundancy

If $\mathbf{A}$ is $(n \times m)$, where $\mathrm{m} < n$, $\rightarrow \ker \mathbf{A} \neq \{\emptyset\}$, and $\forall\, \mathbf{M}_{\mathrm{m}} \in \ker \mathbf{A}$, $\mathbf{A} \cdot \mathbf{M}_{\mathrm{m}} = \mathbf{0}$





Hence, $\forall\, r_{\mathrm{m}} \in \mathbb{R}$, $\mathbf{F}_{\mathrm{m}} = \mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \mathbf{M}_{\mathrm{m}} \cdot r_{\mathrm{m}}$

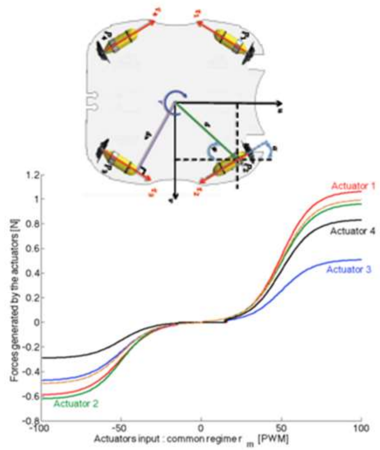
$$\rightarrow \mathbf{F}_{\mathrm{B}} = \mathbf{A} \cdot \mathbf{F}_{\mathrm{m}} = \mathbf{A} \cdot \left(\mathbf{A}^{+} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \mathbf{M}_{\mathrm{m}} \cdot r_{\mathrm{m}}\right)$$

$$= \underbrace{\mathbf{A} \cdot \mathbf{A}^{+}}_{\mathbf{Id}} \cdot \mathbf{F}_{\mathrm{B}}^{\mathrm{d}} + \underbrace{\mathbf{A} \cdot \mathbf{M}_{\mathrm{m}}}_{\mathbf{0}} \cdot r_{\mathrm{m}} = \mathbf{F}_{\mathrm{B}}^{\mathrm{d}}$$

(for some nice properties of A)

$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot \left(\psi^{\mathrm{d}} - \psi\right) - k_2 \cdot r \end{bmatrix}$$

6. Actuation layer robustness



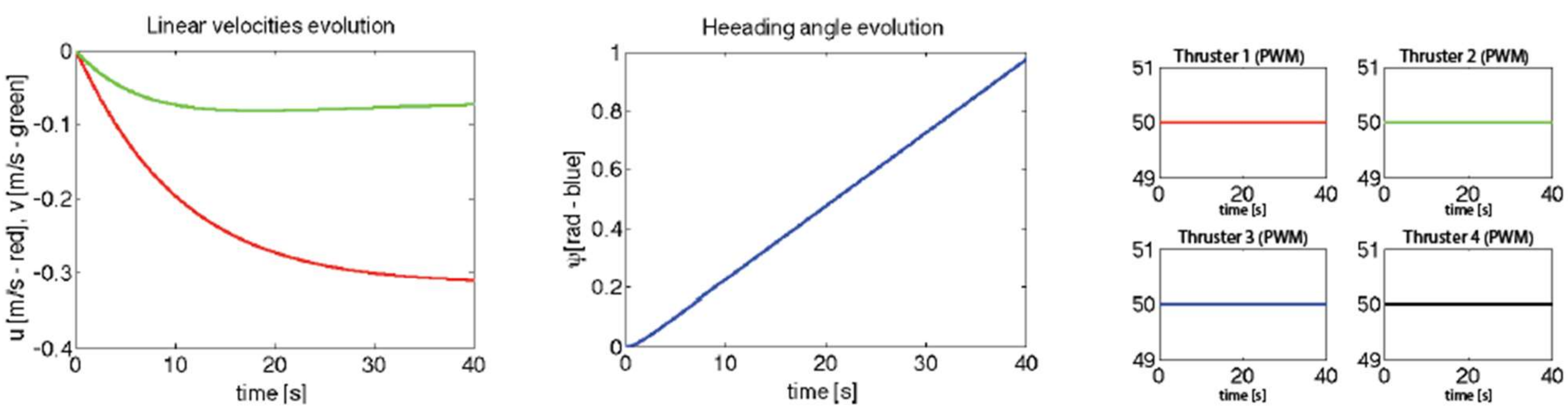
6.a Consider the motors' characteristic uncertainty and disparity

$$\mathbf{F_m} = \Omega(\mathbf{c_m}) \qquad \mathbf{c_m} = \hat{\Omega}^{-1}(\mathbf{F_m})$$

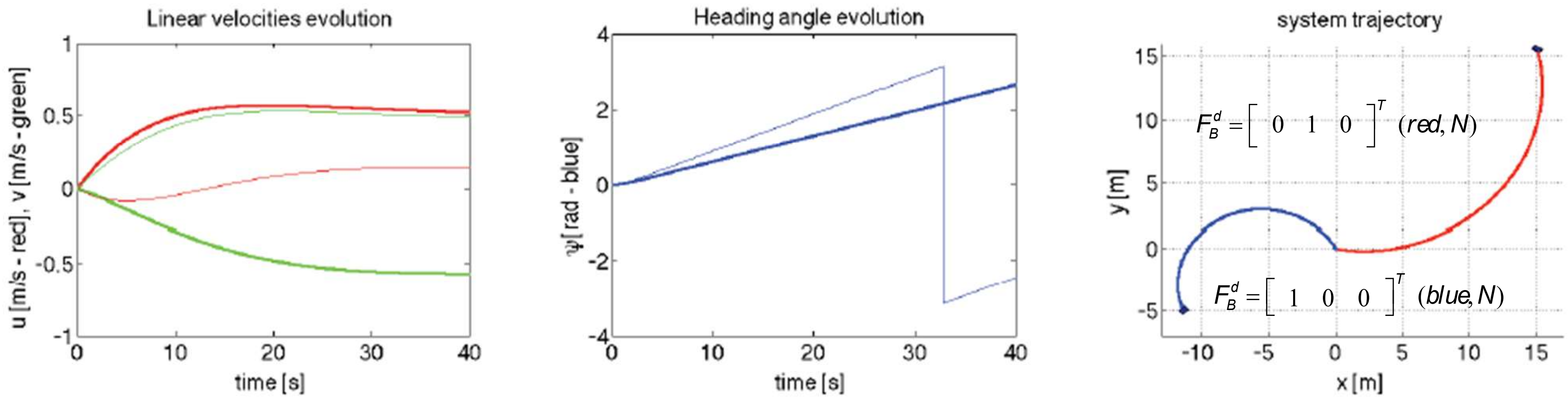
$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A^+}\cdot\mathbf{F_B^d} + \mathbf{M}_\mathbf{m}\cdot r_m\right)\right) \equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A^+}\cdot\mathbf{F_B^d} + \mathbf{M}_\mathbf{m}\cdot r_m\right) \neq \mathbf{F_B^d}$$

➔DOF Coupling effect

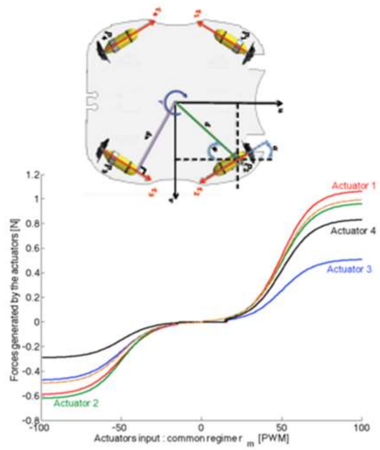
$\mathbf{c_m} = \begin{bmatrix} 50 & 50 & 50 & 50 \end{bmatrix}^T \ (PWM)$



$F_B^d = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
$F_B^d = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$



6. Actuation layer robustness



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➔DOF Coupling effect

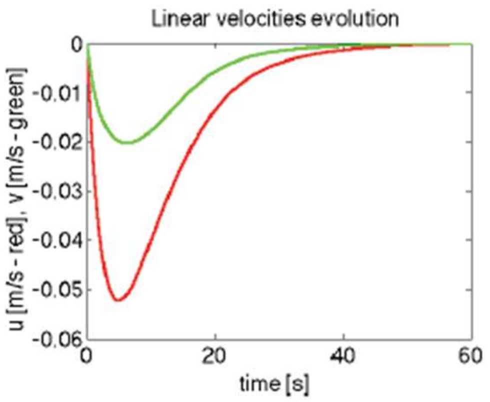
6.b Consider the following closed loop control

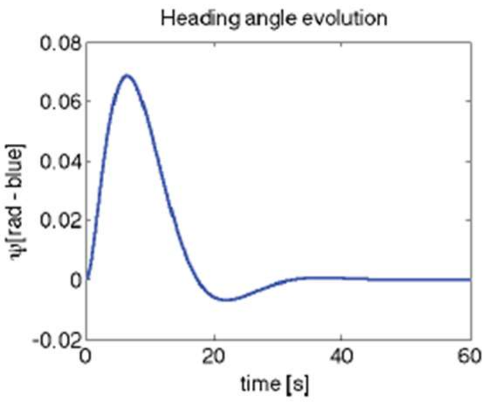
$$\mathbf{F_B^d} = \begin{bmatrix} -u - \int_0^t u\cdot \mathrm{d}t \\ -v - \int_0^t v\cdot \mathrm{d}t \\ -\psi - r - 0.1\cdot\int_0^t \psi\cdot \mathrm{d}t \end{bmatrix}, \begin{cases} \mathbf{c_m} = \hat{\mathbf{\Omega}}^{-1}\cdot\mathbf{A^+}\cdot\left(\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right) \\ r_m = \Omega\cdot c_0 \end{cases}$$

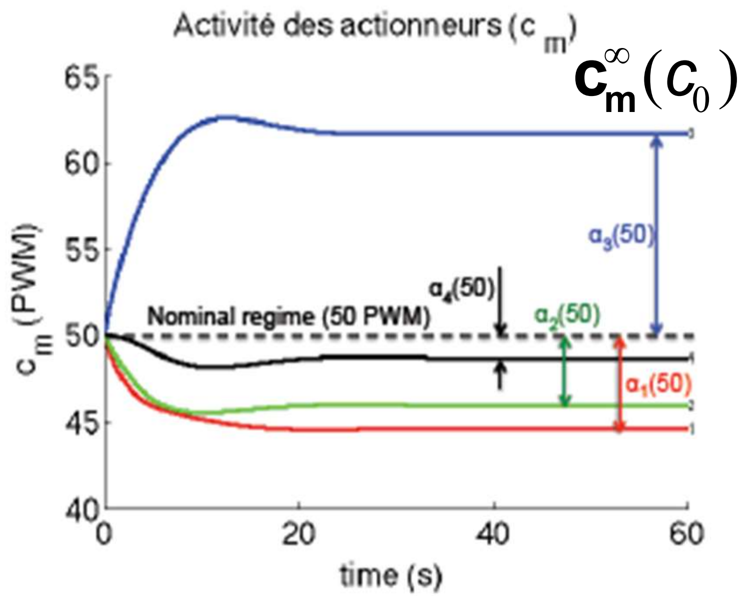
$$\mathbf{F_B} = \mathbf{A}\cdot\Omega\cdot\mathbf{c_m^\infty}(c_0) = \mathbf{0}$$

$$\Rightarrow \mathbf{c_m^\infty}(c_0) \in \ker(\mathbf{A}\cdot\Omega)$$

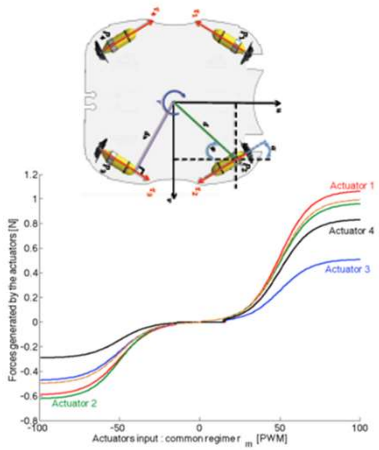
$$\Rightarrow \alpha_i(c_0) = \frac{c_{\mathbf{m},i}^\infty}{c_0}$$







6. Actuation layer robustness



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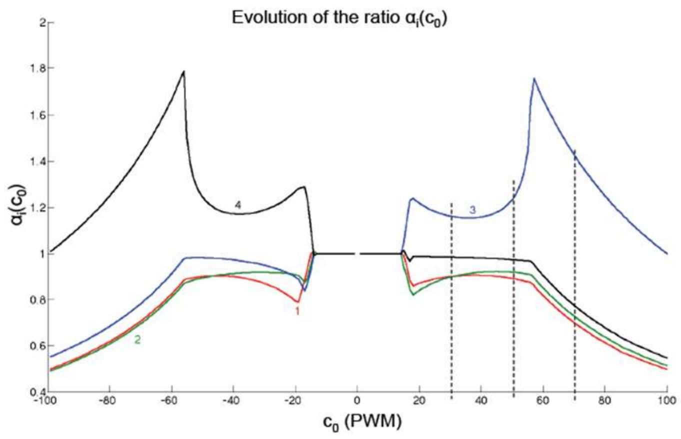
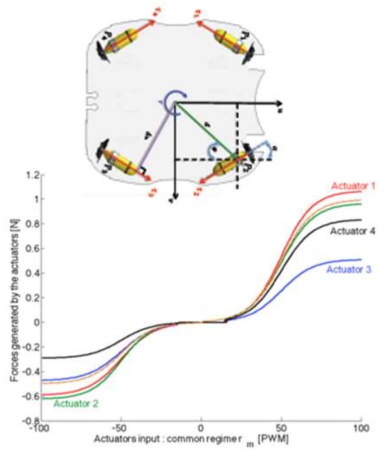
$$\Rightarrow \alpha_i\left(c_0\right) = \frac{c_{\mathbf{m},i}^{\infty}}{c_0}$$

$$\Rightarrow \mathbf{Q}(\mathbf{c_m}) = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

6.c Iterate for $c_{m,\min} < c_m < c_{m,\max}$



6. Actuation layer robustness



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➔DOF Coupling effect

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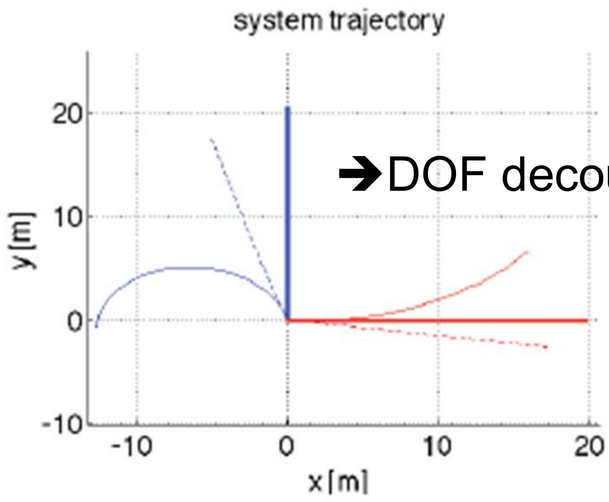
$$\Rightarrow \mathbf{Q}(\mathbf{c_m}) = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

6.c Iterate for $c_{m,\min} < c_m < c_{m,\max}$



➔DOF decoupled

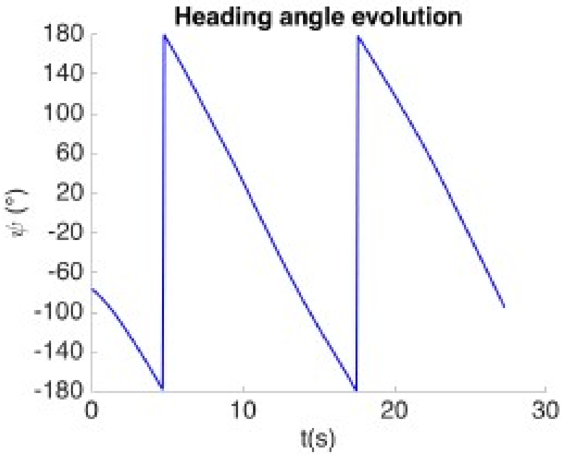
6.d Implement the following open loop co

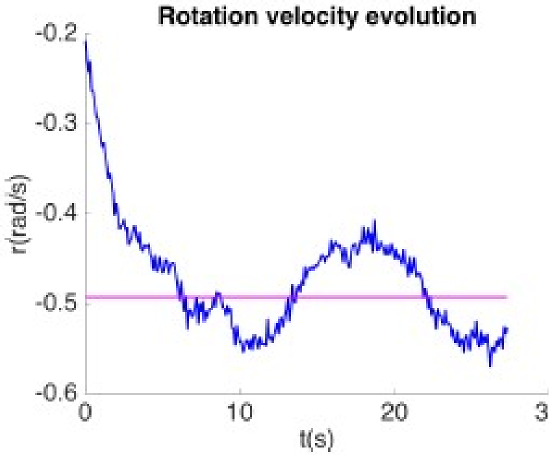
$$\mathbf{c_m} = \mathbf{Q}\left(\hat{\boldsymbol{\Omega}}^{-1}\cdot\mathbf{A}^+\cdot\left(\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right)\right)$$

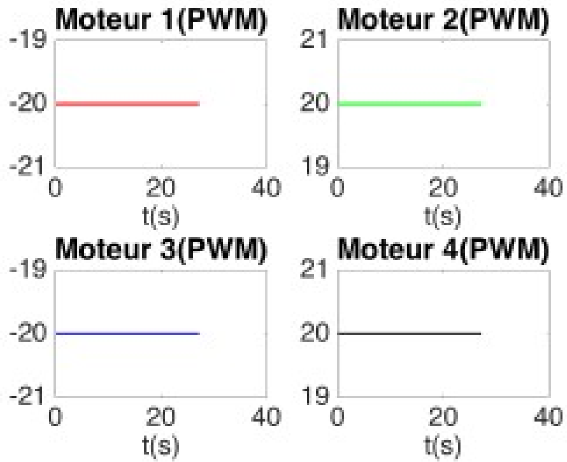
$$\mathbf{F_B} \equiv \mathbf{A}\cdot\boldsymbol{\Omega}\cdot\mathbf{Q}\cdot\hat{\boldsymbol{\Omega}}^{-1}\cdot\begin{bmatrix} \mathbf{A}^+ & M_m \end{bmatrix}\cdot\begin{bmatrix} \mathbf{F_B^d} \\ r_m \end{bmatrix} = k_Q\cdot\mathbf{F_B^d}$$

6. Actuation layer robustness

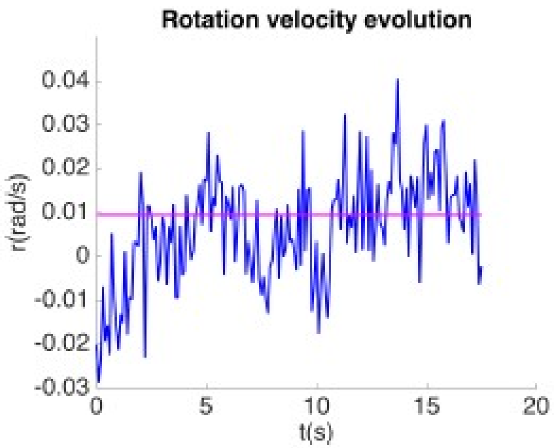
$$\mathbf{c_m} = \hat{\boldsymbol{\Omega}}^{-1} \cdot \mathbf{A}^{+} \cdot \left(\mathbf{0} + \mathbf{M}_{\mathbf{m}} \cdot r_m\right)$$

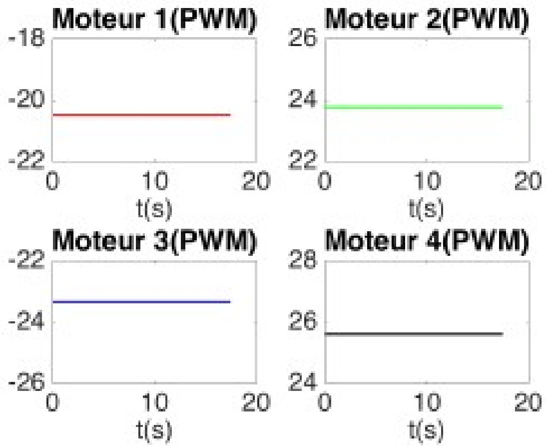


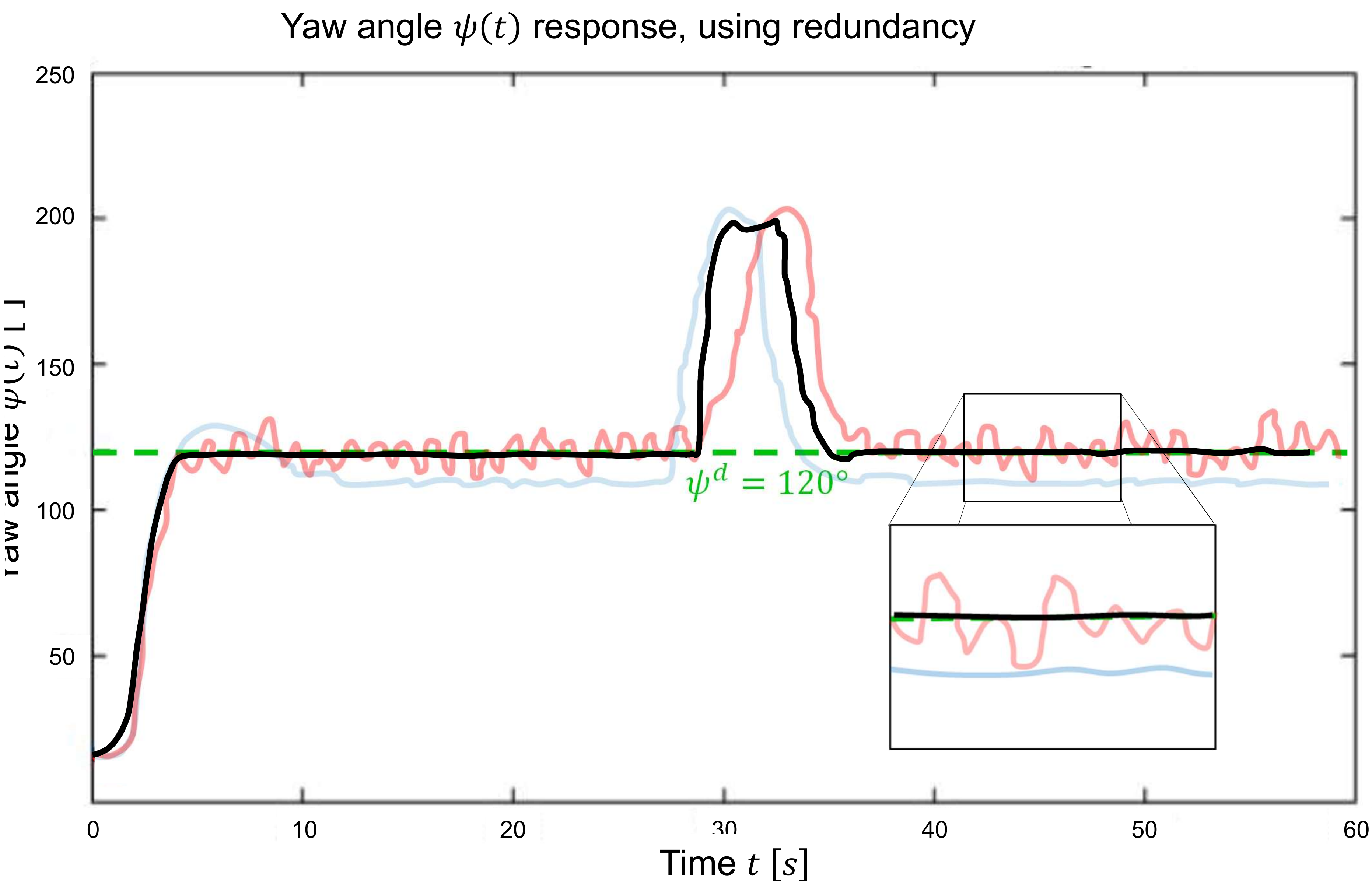




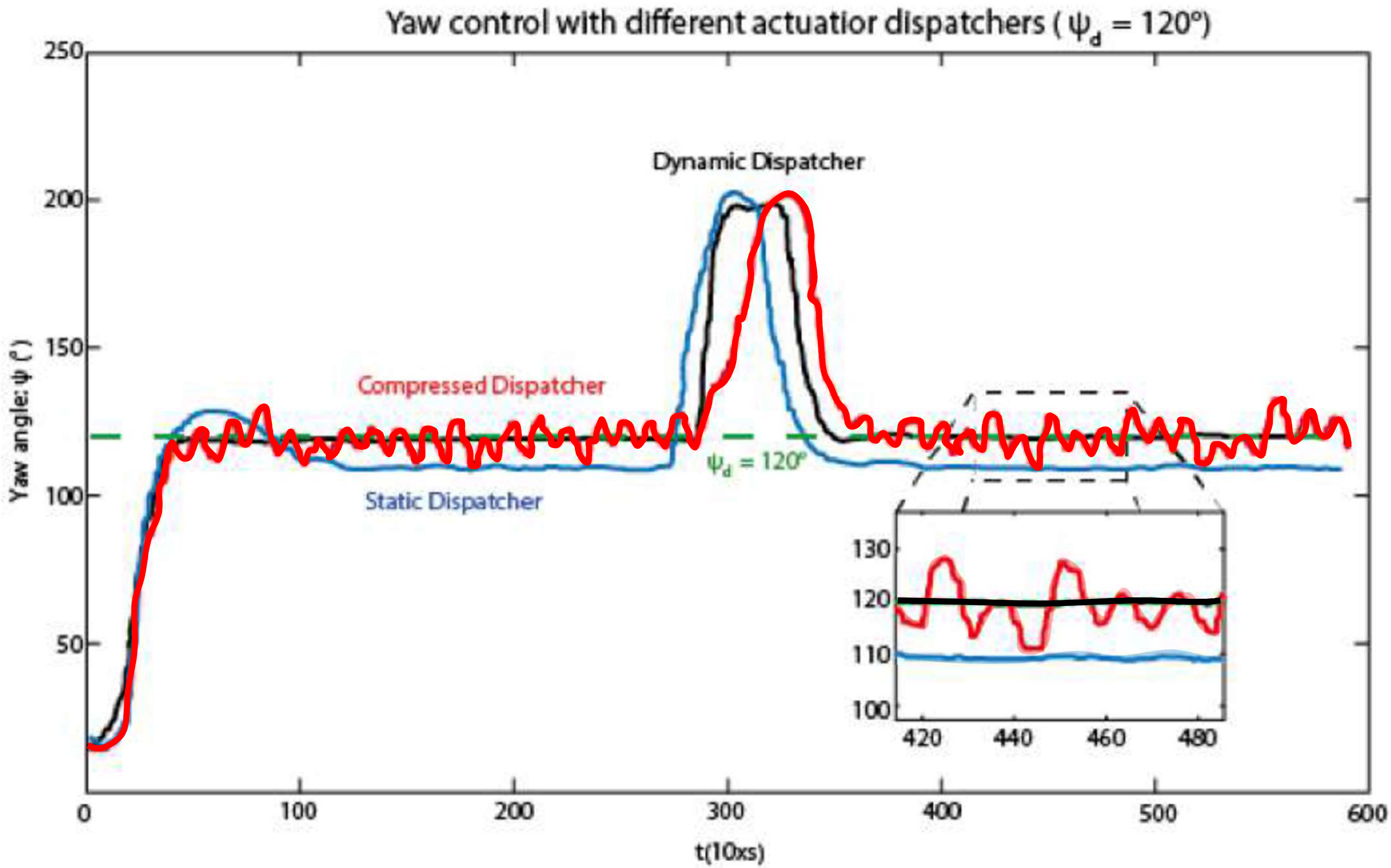
$$\mathbf{c_m} = \mathbf{Q}\left(\cdot \hat{\boldsymbol{\Omega}}^{-1} \cdot \mathbf{A}^{+} \cdot \left(\mathbf{0} + \mathbf{M}_{\mathbf{m}} \cdot r_m\right)\right)$$



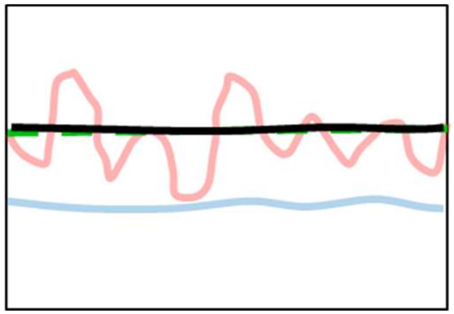


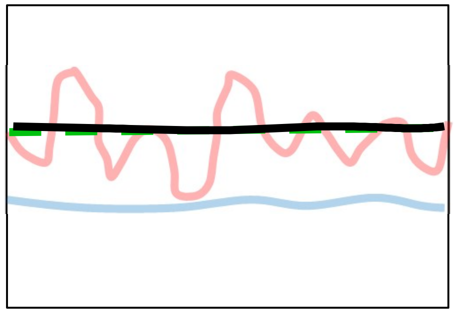


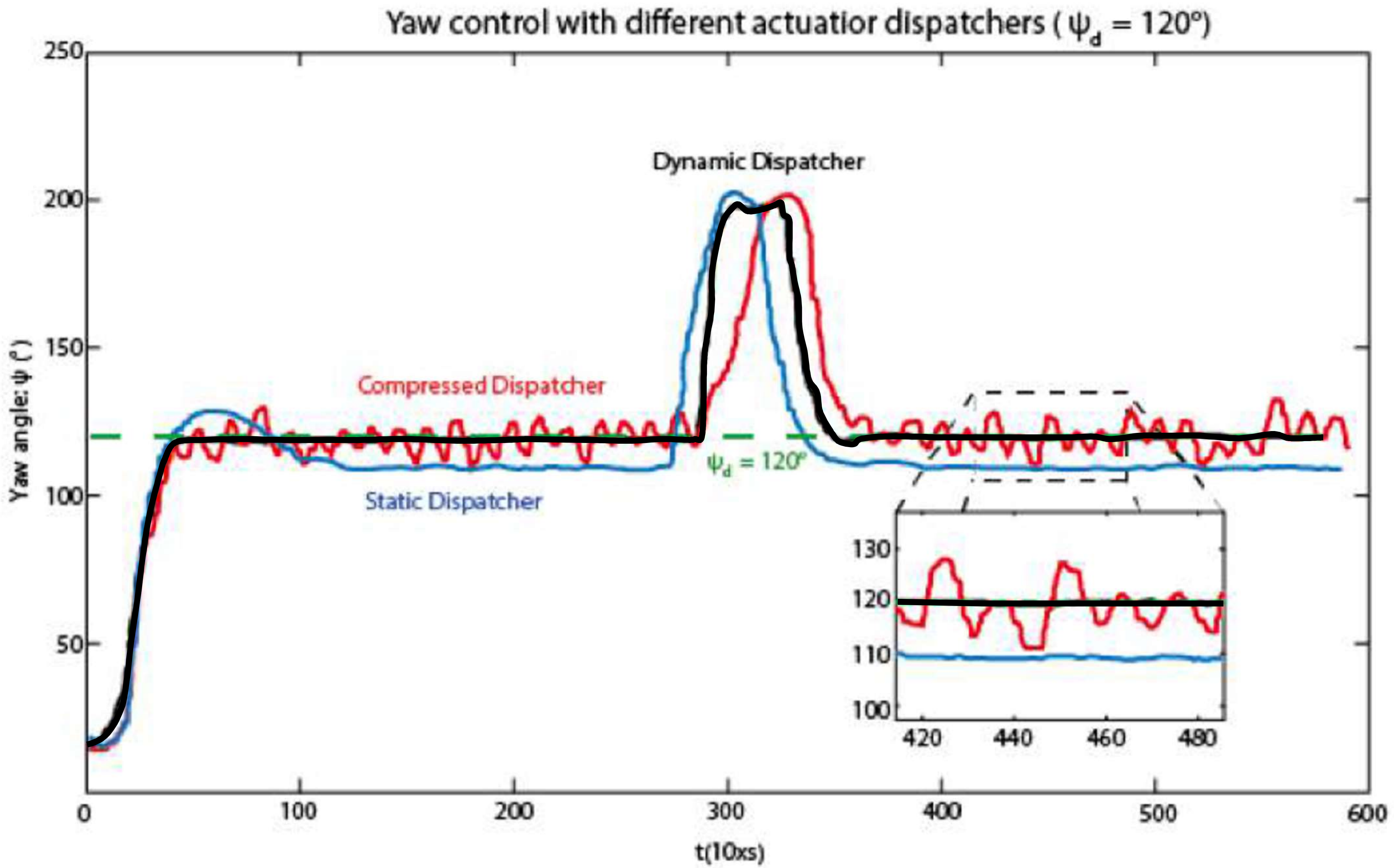
Yaw angle $\psi(t)$ response, using redundancy
250
200
150
100
50
Yaw angle $\psi(t)$ [°]
$\psi^d = 120°$
0
10
20
30
40
50
60
Time t $[s]$



Yaw control with different actuatior dispatchers (ψ_d = 120°)
Dynamic Dispatcher
Compressed Dispatcher
Static Dispatcher
ψ_d = 120°
Yaw angle: ψ (°)
t(10xs)
250
200
150
100
50
0
100
200
300
400
500
600
130
120
110
100
420
440
460
480







Yaw control with different actuatior dispatchers (ψ_d = 120°)
Dynamic Dispatcher
Compressed Dispatcher
Static Dispatcher
ψ_d = 120°
Yaw angle: ψ (°)
t(10xs)
250
200
150
100
50
0
100
200
300
400
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600
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120
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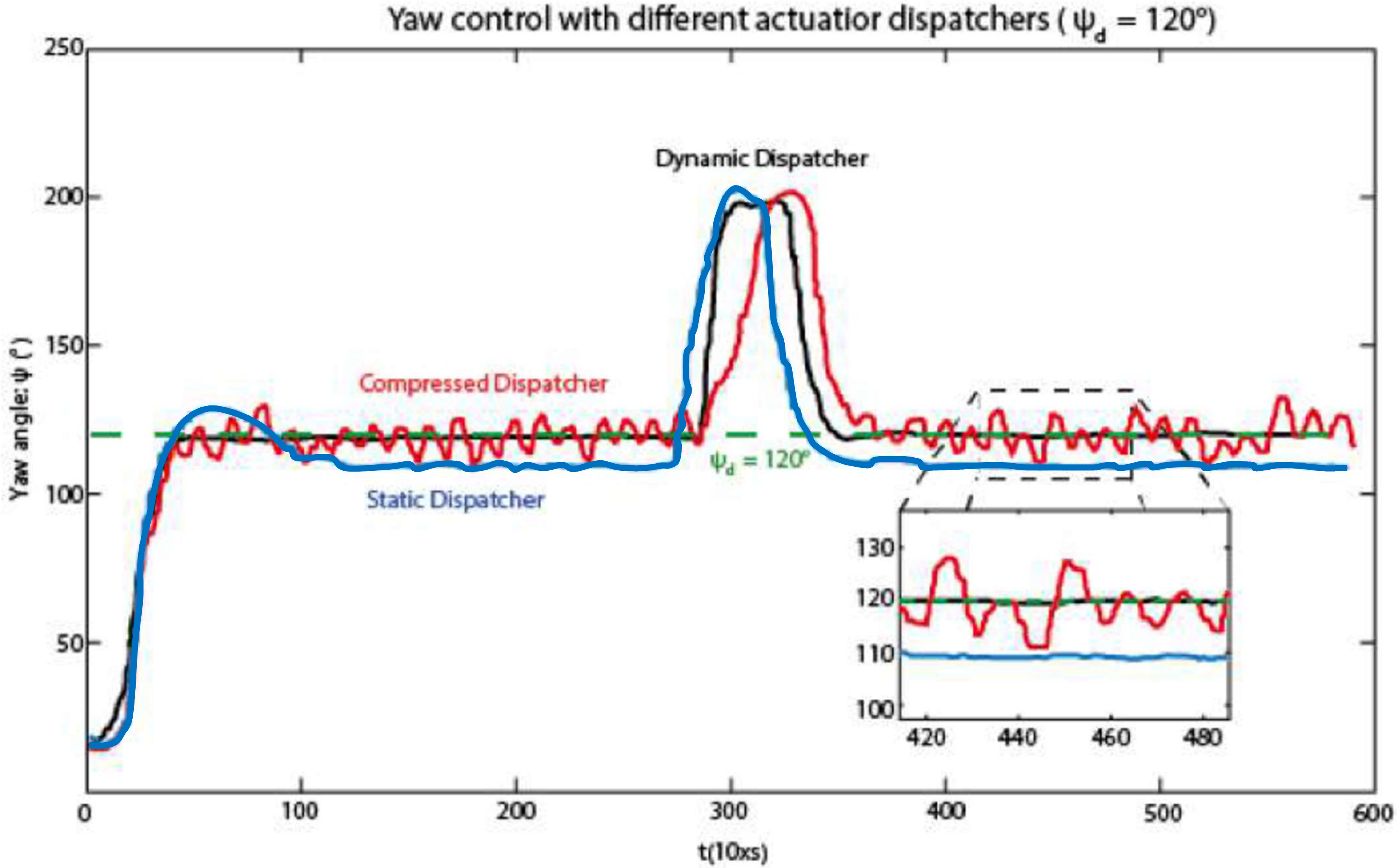
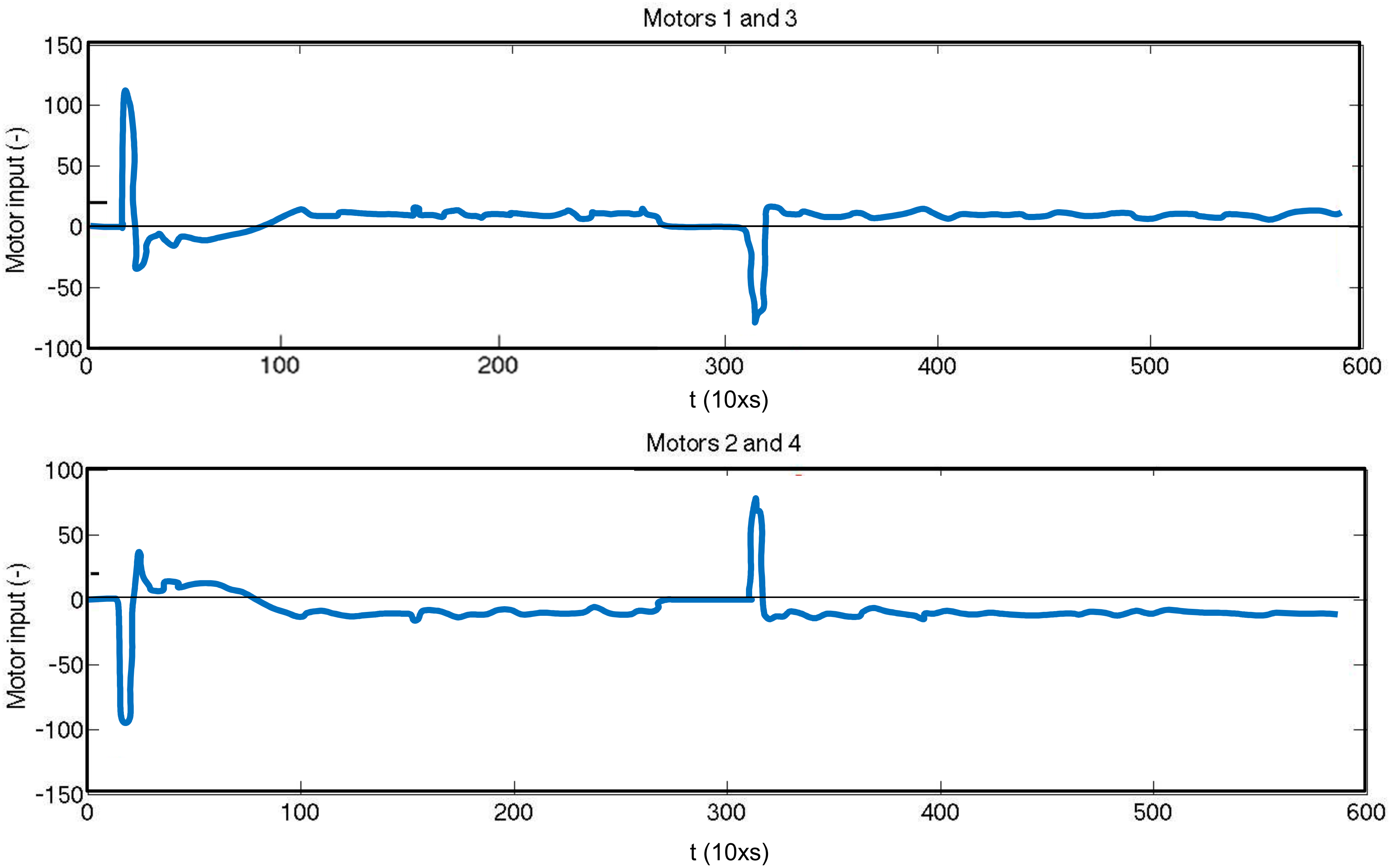
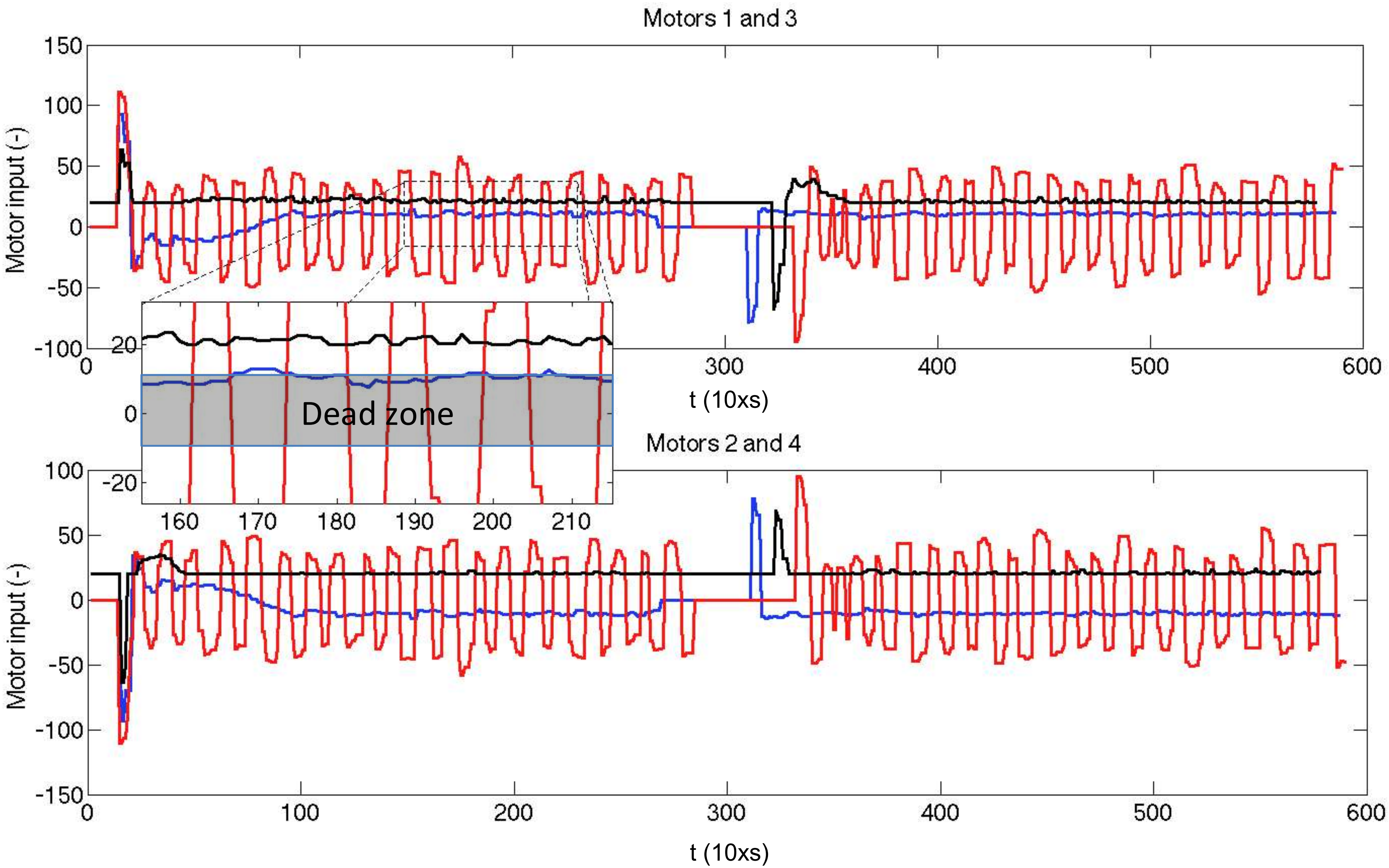
440
460
480



Yaw control with different actuatior dispatchers (ψ_d = 120°)
Yaw angle: ψ (°)
t(10xs)
Dynamic Dispatcher
Compressed Dispatcher
Static Dispatcher
ψ_d = 120°
0
50
100
150
200
250
0
100
200
300
400
500
600
100
110
120
130
420
440
460
480



Motors 1 and 3
150
100
50
0
-50
-100
Motor input (-)
0
100
200
300
400
500
600
t (10xs)
Motors 2 and 4
100
50
0
-50
-100
-150
Motor input (-)
0
100
200
300
400
500
600
t (10xs)

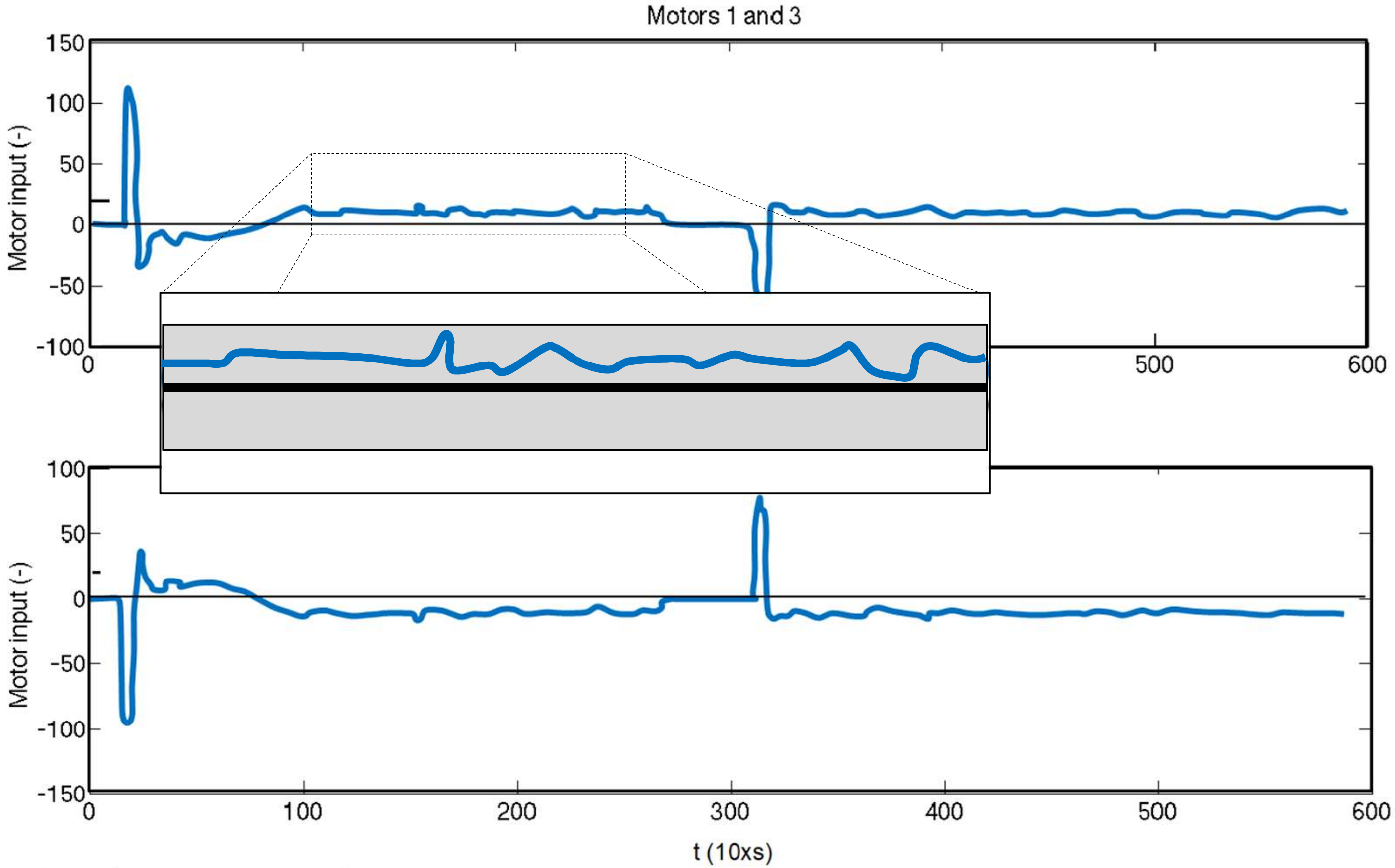

Motors 1 and 3
Motor input (-)
150
100
50
0
-50
-100
0
300
400
500
600
t (10xs)
20
0
-20
160
170
180
190
200
210
Dead zone
Motors 2 and 4
Motor input (-)
100
50
0
-50
-100
-150
0
100
200
300
400
500
600
t (10xs)

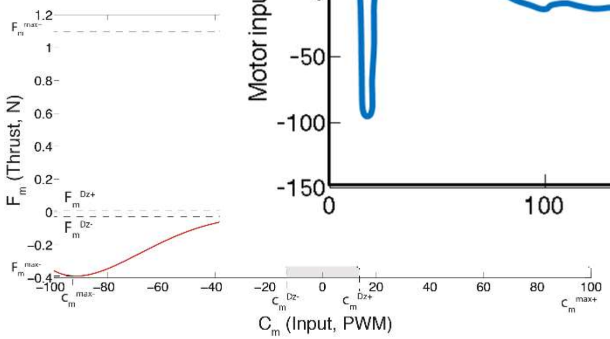
Dead zone

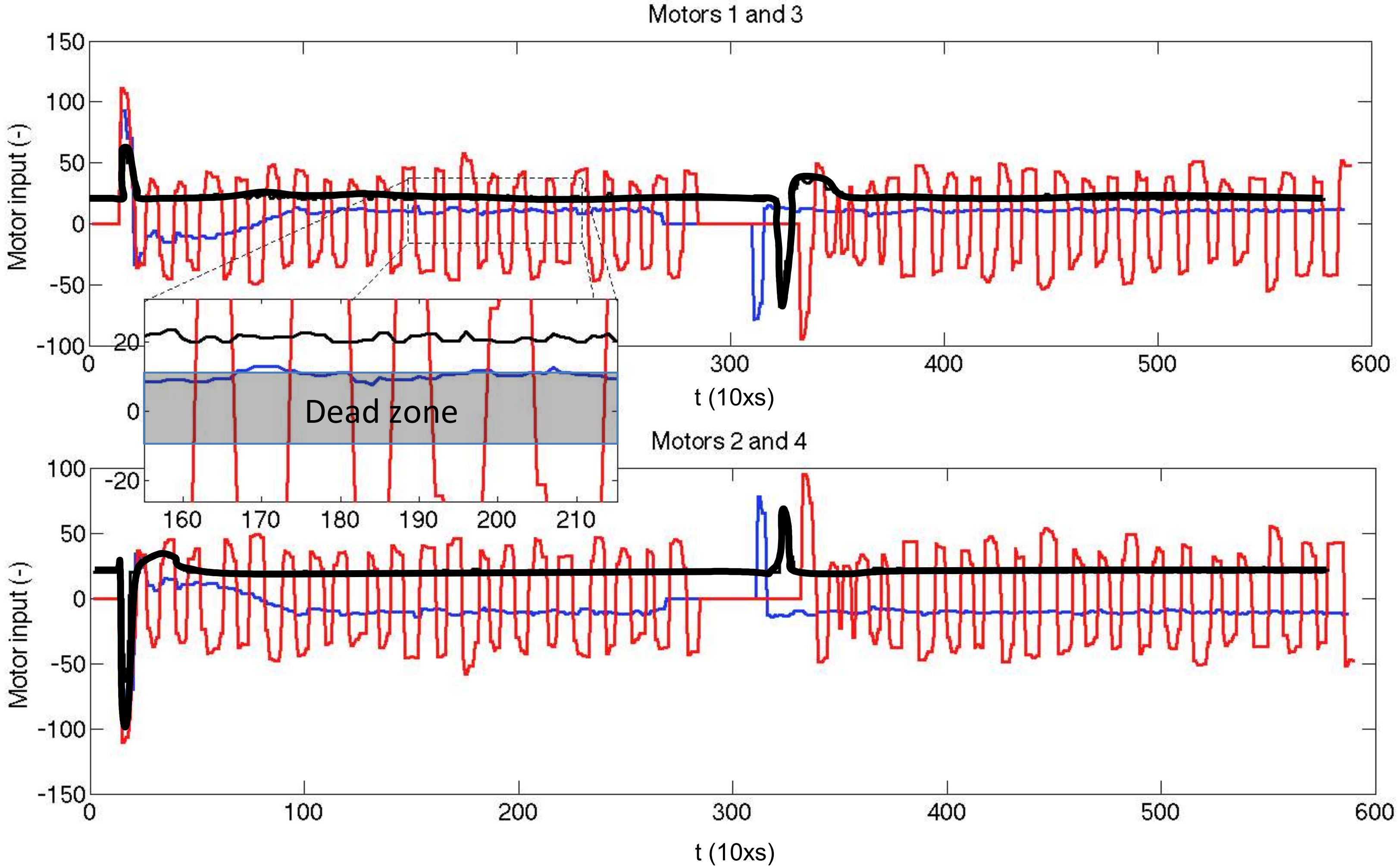
Redundancy

- Example : Jack, classical approach
  - Use of the Moore Penrose pseudo inverse
  - Test o

$$\mathbf{F}_{\mathrm{B}}^{\mathrm{d}} = \begin{bmatrix} F_u^{\mathrm{d}} \\ F_v^{\mathrm{d}} \\ \Gamma_r^{\mathrm{d}} \end{bmatrix} =$$







Motors 1 and 3
150
100
50
0
-50
-100
Motor input (-)
0
300
400
500
600
t (10xs)
20
0
-20
Dead zone
160
170
180
190
200
210
Motors 2 and 4
100
50
0
-50
-100
-150
Motor input (-)
0
100
200
300
400
500
600
t (10xs)